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A REVIEW AND ANALYSIS OF THE MITRE BEACON COLLISION AVOIDANCE S--ETC(U)

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(IDA Study S-481)

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16. Abstract

This report presents the results of IDA's study and analysis of the beacon collision avoidance system (BCAS) design proposed by The MITRE Corporation. This system, referred to herein as MCAS, is an active collision avoidance system which makes use of air traffic control radar beacon system (ATCRBS) transponders already possessed by a large fraction of the flying aircraft population. It is also compatible with the discrete address beacon system (DABS) in anticipation of that system's eventual adoption in place of ATCRBS.

MCAS derives the same economic benefits accruing to any BCAS by virtue of relying on the use of ATCRBS transponders already installed in many aircraft. However, because the system is active, its design must assume, in principle, that airborne MCAS interrogator equipment will be limited to about ten percent of the flying population in order to avoid excessive garble for its own purposes and interference with regular ATCRBS ground stations.

MCAS uses essentially the same threat logic for encounters between MCAS-equipped aircraft which was recommended by ANTC-117 and its performance will therefore suffer the same deficiencies resulting from the use of that logic as predicted in previous IDA studies. These deficiencies are a high natural alarm rate due to a lack of certain parameter data, such as bearing rate and relative acceleration, and/or less than safe alarm criteria resulting from trade-offs intended to reduce the alarm rate. For encounters with ATCRBS transponder equipped aircraft, MITRE has designed a special, so-called remitter logic, which has similar deficiencies.

Because it is active, MCAS tends to be particularly sensitive to high traffic densities which generate excessive amounts of garble for the system. It is apparent, in fact, that even with the most optimistic projection for the effectiveness of proposed garble suppression techniques, MCAS cannot operate successfully in traffic densities as high as half that projected by the Federal Aviation Administration for the Los Angeles Basin in 1982.

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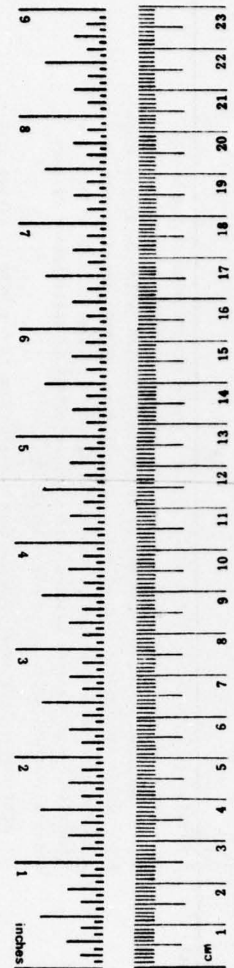
METRIC CONVERSION FACTORS

Approximate Conversions to Metric Measures

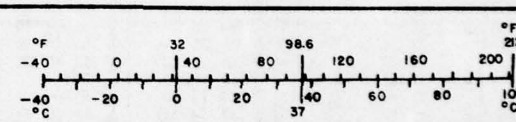
Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
in	inches	*2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km
		AREA		
in^2	square inches	6.5	square centimeters	cm^2
ft^2	square feet	0.09	square meters	m^2
yd^2	square yards	0.8	square meters	m^2
mi^2	square miles	2.6	square kilometers	km^2
	acres	0.4	hectares	ha
		MASS (weight)		
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2000 lb)	0.9	tonnes	t
		VOLUME		
tsp	teaspoons	5	milliliters	ml
Tbsp	tablespoons	15	milliliters	ml
fl oz	fluid ounces	30	milliliters	ml
c	cups	0.24	liters	l
pt	pints	0.47	liters	l
qt	quarts	0.95	liters	l
gal	gallons	3.8	liters	l
ft^3	cubic feet	0.03	cubic meters	m^3
yd^3	cubic yards	0.76	cubic meters	m^3
		TEMPERATURE (exact)		
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C

*1 in = 2.54 (exactly). For other exact conversions and more detailed tables, see NBS Misc. Publ. 286, Units of Weights and Measures, Price $2.25, SD Catalog No. C13.10:286.



Approximate Conversions from Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
mm	millimeters	0.04	inches	in
cm	centimeters	0.4	inches	in
m	meters	3.3	feet	ft
m	meters	1.1	yards	yd
km	kilometers	0.6	miles	mi
		AREA		
cm^2	square centimeters	0.16	square inches	in^2
m^2	square meters	1.2	square yards	yd^2
km^2	square kilometers	0.4	square miles	mi^2
ha	hectares (10,000 m^2)	2.5	acres	
		MASS (weight)		
g	grams	0.035	ounces	oz
kg	kilograms	2.2	pounds	lb
t	tonnes (1000 kg)	1.1	short tons	
		VOLUME		
ml	milliliters	0.03	fluid ounces	fl oz
l	liters	2.1	pints	pt
l	liters	1.06	quarts	qt
l	liters	0.26	gallons	gal
m^3	cubic meters	35	cubic feet	ft^3
m^3	cubic meters	1.3	cubic yards	yd^3
		TEMPERATURE (exact)		
°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	°F



FOREWORD

In this report, the evaluation of the MITRE BCAS has been based on the criteria and specifications, insofar as they are applicable, of ANTC-117. No direct conclusion about the efficacy of the CAS, as such, should be drawn from the fact that the system evaluated herein conforms to the requirements set down by that document. In IDA Study S-450 (FAA Report No. FAA-RD-75-72), an evaluation of ANTC-117, itself, was made. The results of that study indicate that ANTC-117 requirements do not guarantee the safe avoidance of collision between encountering aircraft in all circumstances. Moreover, they predict excessive so-called τ_1 alarm rates in high density traffic such as forecast by the FAA for the Los Angeles Basin in 1982. In another concurrently published paper, IDA Paper P-1215 (FAA Report No. FAA-RD-76-209), there is developed a similar logic with parameters adjusted to provide relative safety for encounters between BCAS-equipped aircraft and those equipped only with an ATCRBS transponder; the latter aircraft does not receive any warnings and does not maneuver to avoid threatening encounters and the logic developed in P-1215 would have higher alarm rates than those predicted in S-450. The proposed MITRE logic is similar to that described in P-1215 but is less conservative with respect to safety. It would, however, have an alarm rate intermediate between that predicted in S-450 and that for the logic in P-1215.

CONTENTS

I. INTRODUCTION

In view of the present and projected state-of-the-art of radar, infrared, and optical sensors, an airborne mid-air collision avoidance system (CAS) design concept, in which a protected aircraft is able to detect all intruder aircraft within its hazard volume, is economically feasible only if it is assumed that the intruders will cooperate actively in the process. In previous studies IDA has analyzed four CAS of the cooperative airborne type.

These systems have been designed, reduced to hardware, and tested. The test results were in general agreement with the analyses presented by the studies, which concluded that any of the systems would perform adequately in accordance with ANTC-117 (Ref. 1) standards in a dense traffic environment such as that projected by the FAA for Los Angeles in 1982 (Ref. 2).

However, all of these systems present a problem which jeopardizes their acceptance by the aviation community. Each requires, in principle, that the entire flying aircraft fleet be equipped with devices, the primary purpose of which is to provide the cooperation demanded by a CAS. It appears likely that much of the general aviation part of the fleet, at least, would regard this as an unreasonable economic burden.

Consequently, the FAA has adopted the philosophy that an acceptable CAS must use existing, already-installed equipment to provide the necessary cooperation and is, in fact, considering for this function the airborne transponders of the existing air traffic control radar beacon system (ATCRBS). Such a beacon CAS (BCAS) would be able to protect an aircraft even

against intruders not equipped with the BCAS, itself, as long as they were equipped with ATCRBS transponders.

Because of possible interference with the ATCRBS, the FAA plans to restrict the installation of the CAS, if adopted, to few classes of users, such as air carriers, and may make its use optional even within those restricted classes. However, if a BCAS which is based on ATCRBS but does not interfere with it can be designed, then its use could be made optional for all potential users.

Development has begun on two BCAS design concepts based on the use of cooperative ATCRBS transponders, one by Litchford Associates and the other by The MITRE Corporation. Of particular advantage to either is the fact that as many as half of the present flying aircraft population are believed to be equipped with appropriate ATCRBS transponders already. Moreover, the fraction of transponder-equipped aircraft is expected to grow, thereby increasing automatically the protection coverage which such a CAS would afford its users in the future. It is clear, of course, that such CAS provide protection against only those aircraft that have suitable ATCRBS transponders. IDA, under contract to the FAA, has completed studies of both of these beacon-type CAS, known collectively as BCAS. This report summarizes the analysis performed on the MITRE version known herein as MCAS. A companion report contains a similar summary of the Litchford version, referred to as LCAS.

The LCAS operation is primarily passive, deriving its basic data from aircraft transponder replies to interrogations by the ground-based ATCRBS secondary surveillance radars (SSR). LCAS processes such data to obtain the altitude, range and bearing of the intruder aircraft whose ATCRBS transponder generated those replies.

The MCAS operates entirely in an active mode, however, and does its own interrogating, eliciting replies in the

standard ATCRBS format from the airborne transponders. MCAS obtains range and altitude, but not bearing, information about an aircraft intruder.

Since LCAS and MCAS are compatible with the ATCRBS, they are subject, although to a different degree, to the problems inherent to the ATCRBS equipment characteristics and specifications, e.g., both BCAS can experience severe mutual interference (garble) between overlapping replies from different transponders. However, because the design of neither BCAS has been completely specified at this time, direct comparisons for the purpose of recommending a selection between them is premature.

Since MCAS is an active system and therefore can exercise some control over communications, it attempts to reduce the amount of garble that it must suffer by operating with a relatively low data rate. By transmitting one interrogation per sec and waiting until a minimum of 30 complete reply signals have been received before an intruder is recognized as a possible threat, it attempts to minimize its own contribution to the stimulation of garble while at the same time assuring that an adequate amount of data are present for the threat evaluation. This must inevitably result in an extension of the time required for recognition of a collision hazard, a problem which MITRE attempts to mitigate by continuous tracking of every intruder from the time it enters the MCAS surveillance volume which extends to a distance of 20 nmi from the MCAS.

Generally, MCAS requires that an alarm be generated by an ATCRBS transponder-equipped aircraft at least 30 seconds prior to a possible collision. Given that under severe garble conditions MCAS needs at least 30 seconds to establish a track on which it can estimate the parameters for threat evaluation, i.e. range, range rate, altitude and altitude rate, the initial four sequential target replies to interrogation that are needed to acquire a track must begin at least 60 seconds before a potential collision. Since it is possible that acquisition of

a track on a target may begin with less than 60 seconds to go, e.g., because the target has just taken off from an airport or has just become visible due to prior masking, MITRE has proposed, but has not as yet developed, a scheme for establishing tracks at acquisition.

The scheme requires that the four initial replies be garble free. Such an idea cannot work in garble conditions in which the full 30 seconds are needed to establish the track, and MITRE has not as yet proposed a solution for this case. Therefore, the problem has not been addressed further in this report.

MCAS obtains the range of an intruder from the time delay of the intruder's reply to its ATCRBS interrogation requesting altitude. MCAS employs ATCRBS mode-C interrogations which all appropriate transponders recognize as requests for replies coded to give the intruder's barometric altitude. The altitude is used along with the range for threat evaluation.

After an intruder track has been established, MCAS uses threat logic similar to, but not identical with, that recommended in ANTC-117 (Ref. 1) for evaluating the hazard potential. This logic is based on the use of range and altitude data alone, since bearing information is not available to the system. It differs in certain respects from that of ANTC-117 primarily because the latter assumes that the intruder is also CAS-equipped and will detect hazards and perform complementary avoidance maneuvers.

MCAS has two modes, one for use with ATCRBS transponders and the other for use with discrete address beacon system (DABS) transponders which are now being developed. When the intruder is equipped with MCAS, or with a DABS transponder, a threat logic more like that of ANTC-117, i.e., one which depends on the intruder's awareness of the threat, is used. This is possible with a DABS-equipped intruder because DABS includes a data link which may permit the MCAS to transmit avoidance instructions to the intruder.

The study which this report summarizes has addressed four problem areas that appear to have the greatest potential impact on MCAS performance from the standpoint of system design. These areas, which are all primarily a result of excessive garble, expected in regions of high traffic density, are: (1) warning delays seriously affecting the system's ability to respond to a threat; (2) false warnings; (3) loss of intruder tracks; (4) saturation of the system's intruder tracking capacity.

Chapter II of this report provides a brief description of ATCRBS, emphasizing those features that are most pertinent to the MCAS operation. A copy of the U. S. Standard for ATCRBS has also been included in this report and appears as Appendix A.

Chapter III contains a brief description of MCAS. The emphasis is on those aspects of the system operation and structure which affect the analysis and performance evaluation presented in later chapters. Hardware considerations are, therefore, not addressed.

Chapter IV discusses quantitatively the factors which affect the MCAS performance. The most important of these are garble and the degarbling techniques that have been proposed to reduce it.

Chapter V presents an analysis of warning delay probability. Chapter VI does the same for false warnings, saturation of intruder tracking capacity and the loss of tracks, all of which are treated essentially as by-products of a process wherein false tracks tend to be established.

Technical details in support of Chapters IV, V, and VI are relegated to appendices which treat in greater depth issues that are discussed in the body of this report.

Chapter VII presents a set of conclusions concerning MCAS performance.

II. DESCRIPTION OF ATCRBS

The Air Traffic Control Radar Beacon System (ATCRBS) is sometimes referred to as the Secondary Surveillance Radar (SSR) since it was originally introduced as a supplement to the Primary Surveillance Radar (PSR), the purpose of which was to provide data pertaining to the location and movement of all aircraft in en route and terminal areas for the benefit of Air Traffic Control. The PSR tracks passive targets, i.e., unaided by beacons, while the SSR obtains tracking data by interrogating beacon transponders aboard the aircraft.

The ATCRBS radar employs two antennas for interrogating aircraft with pulse-coded signals within a range which may vary from one site to another and may be as large as 200 nmi in some locations. One of the antennas is highly directional and transmits a signal for the purpose of eliciting replies from airborne transponders. The other is omnidirectional and transmits a signal, known as the side-lobe suppression (SLS) signal, to provide a reference amplitude which can be used to suppress replies that might be stimulated by the side lobes associated with the directional beam. Both antennas transmit their signals, multiplexed in time, at a frequency of 1030 MHz.

The directional antenna rotates at a rate which varies from station to station. A long-range en route SSR will generally have a slow rotation rate, typically 0.1 rev/sec, while a shorter range terminal SSR will have a faster rate, typically 0.2 rev/sec.

The beamwidth of the directional antenna is specified to be of the order of 3°, although for a fast rotating beam it

may be somewhat larger. The actual sector over which a transponder will reply to the directional antenna signals is determined by their amplitudes relative to the amplitudes of the SLS signals. It is specified, however, that there should be a minimum of 4 to 8 replies to interrogations per main beam passage for each interrogation mode, of which there are two types interlaced in an unspecified periodic pattern (i.e., the pattern may vary from one station to another). It is also specified that aircraft above 15,000 ft must be able to transmit 1200 replies/sec while aircraft below 15,000 ft must be able to transmit 1000 replies/sec for a 15-pulse coded reply and, further, that the maximum interrogation frequency of any SSR shall be 450 interrogations per second.

Three pulses, designated in the sequential order of their transmission as P_1, P_2 and P_3, form the interrogation signal. The first, P_1, is transmitted by both antennas. The second, P_2, is the control pulse transmitted only by the omnidirectional antenna 2 microsec after the P_1 pulse.* The third, P_3, is transmitted only by the directional antenna.

The maximum power in the main beam of the directional antenna is recommended to be at least 24 dB above that in the strongest side lobe, and the power of the signal transmitted

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*Originally SSRs had only the directional antenna and transmitted only the P_1 and P_3 pulses, but as the number of SSRs and transponders increased the system encountered severe self interference (fruit) problems because some transponders were replying to side lobes. Subsequently the omnidirectional antenna was added and the P_2 pulse was transmitted by this antenna in order to eliminate such unwanted replies.

Some military SSRs still transmit only the P_2 pulse on the omnidirectional antenna; however, all FAA and some military SSRs now transmit both the P_1 and P_2 pulses on the omni SLS antenna, a procedure referred to as improved side lobe suppression (ISLS). At present if an SSR has ISLS and has its directional antenna mounted on the same rotating structure as the PSR, with its outputs integrated in a single combined processor, it is said to have integrated ISLS (IISLS or I^2SLS).

by the omnidirectional antenna is supposed to be at least 9 dB (typically 17 dB) below the main beam maximum. Thus, an aircraft can determine whether it is receiving a main beam or a side lobe signal by comparing the amplitudes of P_1 and P_2. The transmission of P_1 by both antennas is intended to aid in discriminating against interrogation signals which are multipath reflections from buildings or other structures rather than direct line of sight transmissions.*

The time interval between the P_1 and P_3 pulses can have any of six discrete lengths: 3, 5, 8, 17, 21, or 25 microsec, and it is used as a code to designate any of six corresponding modes labeled 1, 2, 3/A (usually designated simply as A), B, C, and D. Two of the modes are relevant to Air Traffic Control (ATC) and the CAS application: A, which requests identification (ID), and C, which requests altitude. For the purpose of this report all other modes will be ignored.**

Reply signals are transmitted at a frequency of 1090 MHz in a format consisting of two framing pulses, designated as F_1 and F_2, spaced 20.3 microsec apart and 13 possible pulses in between spaced in increments of 1.45 microsec. The pulse at the center of the 13 pulses between the framing pulses is designated as X and is reserved for future use. The other 12 are used for a 4096 character code to provide identity after a mode A interrogation. Eleven of the 12 pulses are also used to provide altitude in 100-ft increments after a mode C interrogation.

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*Cf. previous footnote.

**Some old transponders still in use provide neither coded ID (mode A) nor altitude (mode C) replies. Other old transponders still in use provide only 64 possible mode A ID codes and no altitude data. Most transponders now in use provide 4096 mode A ID codes and no altitude data. All air carriers and most military and business aircraft use the complete 4096 code mode A and mode C transponders. Under some circumstances military aircraft might not reply to mode A or mode C interrogations. Modes 1 and 2 are primarily for military purposes while B and D are intended for international use.

In addition to the 12 regular pulses used for identification after a mode A interrogation, a Special Position Interrogation Pulse (SPI) may be transmitted 4.35 microsec after the second framing pulse. The SPI is usually transmitted with manual control by the pilot on voice communications request of Air Traffic Control to aid in identifying a particular aircraft which may have been assigned a general rather than a specific identity number or whenever any ambiguity in identifying a return occurs.

The altitude replies after a mode C interrogation are encoded on 8 of the 13 internal reply pulse positions in a Gray code providing barometric altitudes in 500-ft increments from -1000 ft to 127,000 ft. The 500-ft interval is then decomposed into 100-ft intervals by a second Gray code, using three of the remaining five internal reply pulse positions. Two pulse positions, the X and one other, are not used in coding altitude.

The transmission time of a reply is determined by the range propagation delay of a valid P_3 pulse, i.e., one from a P_1, P_2, P_3 reply sequence for which the relative pulse amplitudes are in a relationship appropriate to a main beam transmission. Thus, when the SSR interrogator receives the transponder reply delayed again by the range propagation it can determine the range to the transponder from a measurement of the total time between the transmission of the P_3 pulse and reception of the first framing pulse in the reply (which it recognizes by the 20.3 microsec interval between the F_1 and F_2 pulses) after removing the fixed calibrated delays of the transponder. The SSR also estimates the azimuth of the transponder by beam splitting on all of the replies that have the same range delay.

Each SSR in a particular region transmits a different interrogation sequence pattern which can be used to identify it. The interrogation rate of any SSR is required to differ from that of any other by at least 5 interrogations/sec. In addition, some transmit at a fixed message repetition interval

(PRI) while others transmit in one of several possible staggered patterns. One of the latter is referred to as jittered PRI since it uses three PRIs, the smallest of which differs from the next smallest by the same amount as that one differs from the largest. In the other staggered patterns the periodic pulse train has a cycle of interpulse spacings consisting of as many as seven different intervals in successive transmissions.

Not all aircraft are presently equipped with ATCRBS transponders nor is it expected that they will be by 1985. Of the aircraft which are so equipped, not all have transponders capable of replying in mode C, transmitting altitude data, nor is it expected that all transponders will have such capability by 1985.

III. DESCRIPTION OF THE MITRE BCAS

A. GENERAL CONCEPT

The MITRE version of BCAS (MCAS) is an active cooperative mid-air collision avoidance system which makes use of transponders already possessed by the encountering aircraft as part of the ATCRBS (or DABS). The MCAS is independent of ground installations but depends upon the automatic response by airborne transponders to coded interrogations. The MCAS operation is active in that MCAS transmits the coded interrogations which stimulate the ATCRBS or DABS transponder replies. Those replies are used by MCAS for range-tracking intruder aircraft equipped with appropriate transponders.

In the ATCRBS mode MCAS interrogates with mode-C signals, requesting barometric altitude. Thus, protection is afforded only from those aircraft which possess transponders capable of replying to ATCRBS mode-C interrogations (or to DABS). At present, probably as much as half of the usual total active air traffic at any given time is equipped with transponders having mode-C capability. FAA regulations require that all aircraft utilizing some high traffic density, controlled airports and all aircraft flying above 12,500 ft must be equipped with transponders having mode-C response capability. As for MCAS, itself, MITRE postulates, as a necessary requirement imposed by system design considerations, that MCAS will be restricted to no more than 10 percent of the flying traffic in a high-density terminal area such as Los Angeles.

Figure 1 is a simplified block diagram of MCAS. The system has been designed to be compatible with ATCRBS and DABS

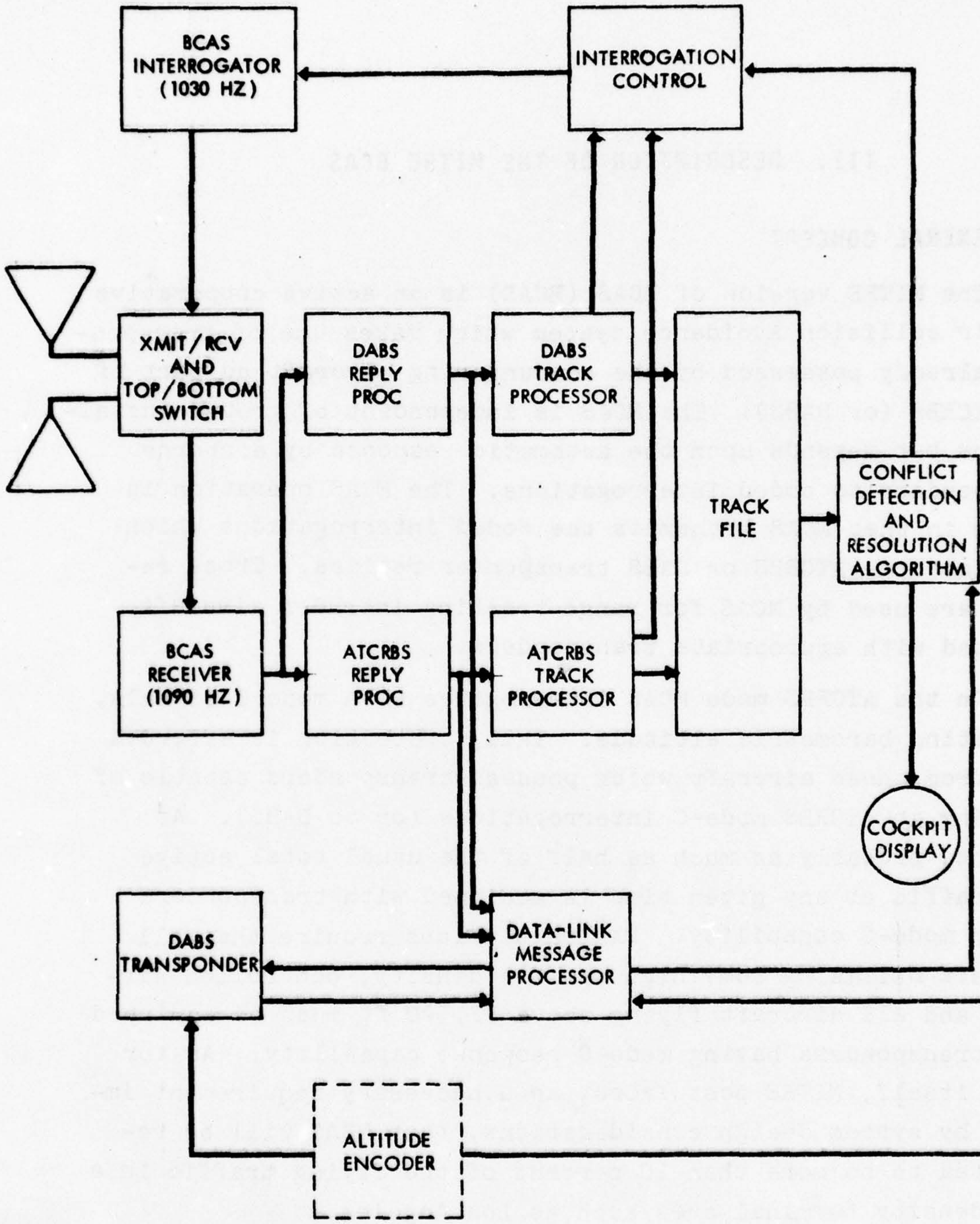


FIGURE 1. Active BCAS Block Diagram

transponders. The most severe garble problem exists with the ATCRBS transponder operation and most of the attention in the MCAS design as well as this report is concerned with the ATCRBS mode of operation. However, in the interest of providing the reader with some information on the DABS mode of operation, a minimum description of that mode is also included here.

B. THE DABS MODE

The MCAS is designed to include a DABS transponder and interrogator as an essential element. DABS transponders in an environment in which they are being interrogated by ground DABS interrogators will be transmitting replies that are encoded with the DABS transponder discrete address, i.e. identity. In addition, in order to accommodate BCAS needs, the DABS transponder specification has been modified to require the transponder to generate spontaneous replies at a random low repetition rate (about 1 Hz) in the absence of valid interrogations. These random replies are called "squitter." The MCAS receiver processes such DABS transponder messages (squitter and those stimulated by all interrogators) and stores the altitude and identity of the transponder. Generally, such signals from all DABS transponders will be at a sufficiently low duty cycle so that garble will be at relatively low levels and the MCAS can receive such signals reliably. The format and redundancy of the DABS signals have been designed to operate simultaneously in the presence of ATCRBS signals.

After the signals of a DABS transponder have been stored in the track file and tracked in altitude, the MCAS will interrogate the specific DABS transponder using its discrete address identity code if it appears to represent a threat, based only on altitude data. Simple altitude threat criteria are used: either the transponder's altitude is within prescribed limits, e.g., within ±3,000 ft of the MCAS altitude, or the transponder has an altitude and altitude rate such that it is predicted to

reach the MCAS altitude within a prescribed time limit, e.g., 60 seconds. From the DABS transponder reply to an MCAS interrogation, the range of the transponder can be determined. If the range is too great to represent a current threat, e.g., greater than 10 nmi, then subsequent interrogations will proceed at a low rate, e.g., once per 10 seconds and a range track correlated with identity and altitude data will begin and will be stored in the file. As the encounter proceeds, if the transponder range decreases or the track appears threatening, the interrogation rate will increase.

The MCAS will use the full ANTC-117 (Ref. 1) logic in evaluating DABS threats. This logic depends on both aircraft performing complementary vertical escape maneuvers. Since the DABS transponder includes a data link and display for other purposes, the MCAS can transmit the desired maneuver for the transponder. If both aircraft are MCAS-equipped and have separately evaluated the threatening condition, the appropriate maneuvers for each are determined by mutual data exchange on the data link.

C. INTERROGATION AND REPLY IN THE ATCRBS MODE

In the ATCRBS mode of operation each MCAS transmits a programmed group of mode-C (altitude request) and suppression interrogations omnidirectionally at a rate of one Hz. The programmed group is used to reduce the effects of garble to a level that is acceptable to the tracker and will be discussed more fully later. For simplicity the MCAS will be described as if it only transmitted mode-C interrogations, and when garble reduction techniques are discussed the use of suppression interrogations will be explained.

Each mode-C interrogation consists of a pair of 0.8μs duration pulses (designated P_1 and P_3) separated by 21μs; each suppression interrogation consists of a pair of 0.8μs pulses (designated P_1 and P_2) separated by 2μs. Any ATCRBS transponder

with mode-C capability that receives a valid mode-C pulse pair, replies with an altitude pulse-coded message that is transmitted 3.0 ±0.5μs after receipt of the P_3 pulse.

The reply consists of two framing pulses (F_1 and F_2) separated by 20.3μs, plus a number of internal pulses positioned in accordance with the ATCRBS code for the barometric altitude of the transponder. All reply pulse shapes are identical, within equipment tolerances, and all reply pulse widths are nominally 0.45μsec.

Between the F_1 and F_2 pulses there is space for thirteen pulses with an interposition spacing of 1.45μs. Any pulse detected at a prescribed pulse position is decoded as a binary "one," while the absence of a pulse is decoded as a binary "zero." The complete altitude pulse code format is shown in Fig. 2. Only eleven of the thirteen possible pulse positions are used for the altitude code; the center position (designated X) is not used at present, but is reserved for future use; the thirteenth position, while not used for altitude encoding, is used by the ATCRBS for identity and other modes that are not part of the MCAS concept as presently planned.

D. REPLY DETECTION

The two reply framing pulses, or bracket pulses, are detected by a bracket detector (part of the MCAS ATCRBS reply processor, cf. Fig. 1) which provides a binary "one" output for every pulse pair whose pulses are separated by 20.3μsec. Upon detection of such a pulse pair, all other pulses detected at the prescribed altitude pulse positions within the bracket are converted into a binary sequence. This altitude code pulse sequence, together with its bracket pulses, forms the basic reply input data to the track processor or tracker.

Bracket detection is accomplished as follows. A reply waveform which is intercepted by the MCAS antenna will be amplified and band-limited in the receiver. The signal is then fed

ALTITUDE TRANSMISSION CODE

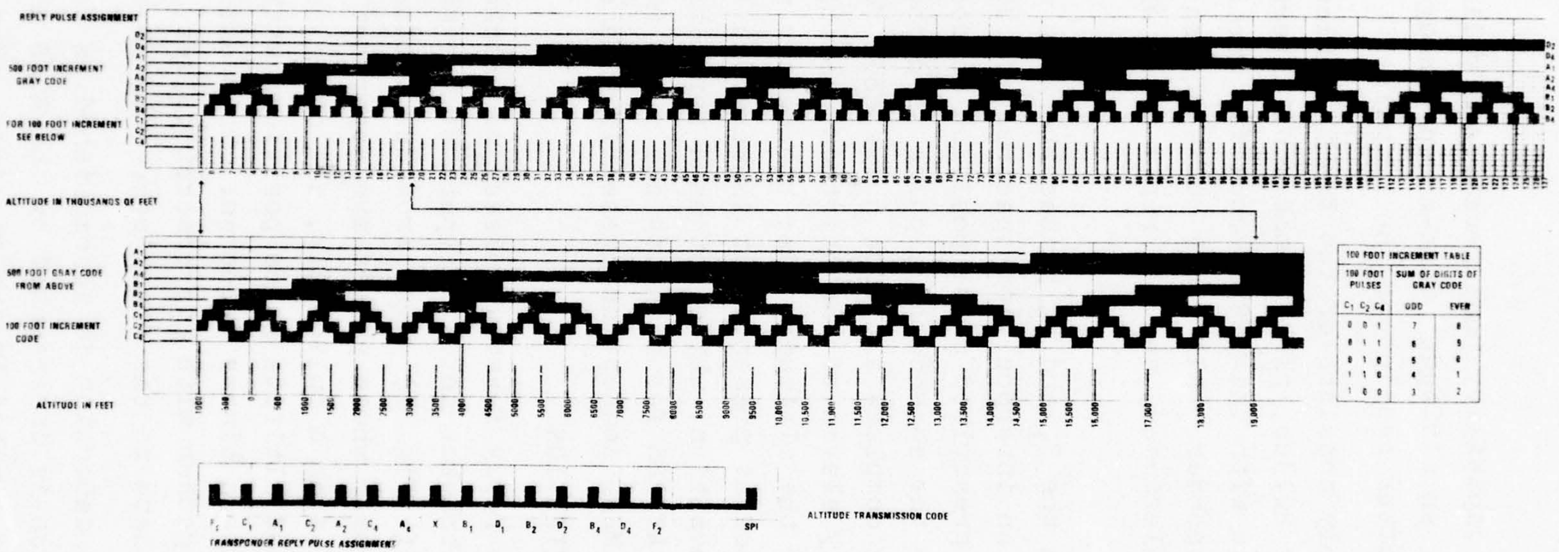
UNIT DISTANCE REFLECTED BINARY CODE FOR 8 BITS

A4, B1, B2, B4 PULSES* (columns); D2, D4, A1, A2 PULSES* (rows)

	0000	0001	0011	0010	0110	0111	0101	0100	1100	1101	1111	1110	1010	1011	1001	1000
0000	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0001	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16
0011	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47
0010	63	62	61	60	59	58	57	56	55	54	53	52	51	50	49	48
0110	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79
0111	95	94	93	92	91	90	89	88	[illegible]	86	85	84	83	82	81	80
0101	96	97	98	99	100	101	[illegible]	103	104	105	106	107	108	109	110	111
0100	127	126	125	124	123	122	121	120	119	118	117	116	115	114	113	112
1100	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143
1101	159	158	157	156	155	154	153	152	151	150	149	148	147	146	145	144
1111	160	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175
1110	191	190	189	188	187	186	185	184	183	182	181	180	179	178	177	176
1010	192	193	194	195	196	197	198	199	200	201	202	203	204	205	206	207
1011	223	222	221	220	219	218	217	216	215	214	213	212	211	210	209	208
1001	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239
1000	255	254	253	252	251	250	249	248	247	246	245	244	243	242	241	240

256 INCREMENTS (500 FOOT EACH)
GIVING ALTITUDE FROM -1000 FEET TO 127,000 FEET

*0 or 1 in a pulse position indicates the absence or presence of a pulse respectively.



100 FOOT INCREMENT TABLE		
100 FOOT PULSES C_1 C_2 C_4	SUM OF DIGITS OF GRAY CODE ODD	EVEN
0 0 1	7	8
0 1 1	6	9
0 1 0	5	0
1 1 0	4	1
1 0 0	3	2

FIGURE 2. Altitude Code Format

(Extracted from "U.S. National Aviation Standard for the Mark X (SIF) Air Traffic Control Radar Beacon System (ATCRBS) Characteristics," DOT/FAA Selection Order, 8 March 1971).

to an analog detector and to a digital quantizer where it is converted to a logic format (i.e., a "1" or "0") for use by the bracket pulse decoder. A block diagram of the bracket pulse decoder is shown in Fig. 3. The bracket pulses of a reply waveform are separated in time by 20.3μsec. The equipment checks for pulses with this time separation by comparing the quantized video with a delayed replica of itself. A 168-bit shift register is used to produce the desired delay. The shift register operates at a clock rate of 8.276 MHz so that each bit corresponds to a time duration or a delay of 121 nsec.

The input and output of the shift register are combined in a 2-input logical "and." This circuit produces a logic 1 output only when the two input signals are both logic 1's. The quantized video input signals are at the logic 1 level when the receiver's MDS threshold is exceeded, so that the operation of the 2-input "and" checks for received pulses with the correct time separation.

The output of the 2-input "and" feeds a 2-bit shift register and one input of a 3-input "and." The 2-bit shift register operates at the same clock rate as the 168-bit shift register discussed above. The 2-bit shift register produces two outputs which are a delayed replica of the input signal. The replica which is delayed by 121 nsec (i.e., one bit), is used as the second input to the 3-input "and", and the replica which is delayed by 242 nsec feeds an inverting input to the 3-input "and." A logic 1 at the output of the 3-input "and" constitutes a bracket pulse detection and causes the range counter to be read and the presence of altitude code bits to be checked.

A bracket pulse detection occurs when a logic 0 appears at the inverting input and a logic 1 at the two non-inverting inputs to the 3-input "and." In other words, the data stream output from the 2-input "and" must consist of a logic "0"

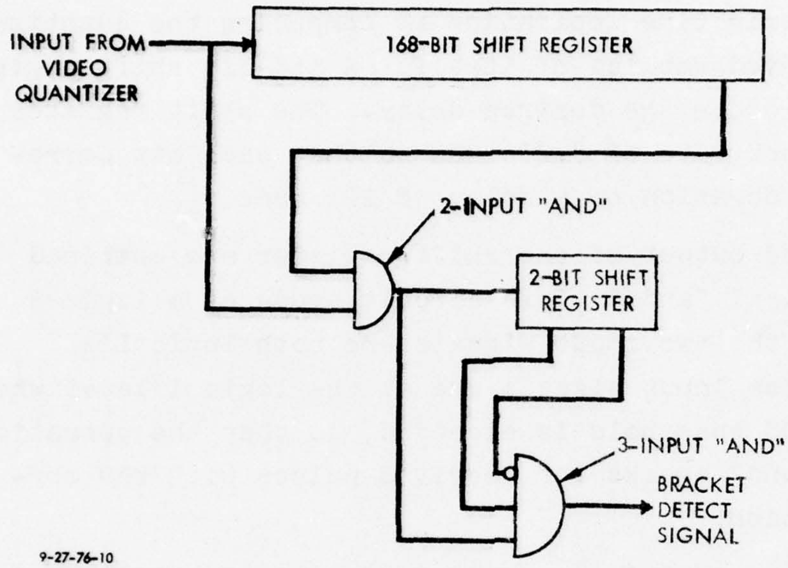


FIGURE 3. Bracket Pulse Detection Decoder

followed by two logic 1's. Working backwards a logic 0 occurs at the output of the 2-input "and" if either the delay or undelayed quantized video is at a logic 0 level. This in turn requires that the received signal level fall below the receiver's MDS threshold. A bracket pulse detection therefore occurs only if the received signal level drops below the threshold immediately preceeding one or both of the received reply message bracket pulses.

E. TRACKING

1. Track Initiation

Tracks are formed by processing stored reply sequences from four successive interrogations at 1-sec intervals. The starting points, at any given time, are two-second-old replies. For any one such reply (see Fig. 4), all one-second-old replies are examined whose apparent range has closed by 0 to 1650 ft (a maximum closing rate of 977 knots) from the two-second-old replies. Figure 4 shows two such one-second-old replies that satisfy this criterion and one indicated by the circle that fails the test. For each two-second-old reply, all one-second-old replies are tested. Reply pairs formed from two-second-old replies and associated one-second replies meeting the closing range criterion (any one-second-old reply may be paired off with more than one two-second-old reply and vice versa) form an initial reply pair which is used to extrapolate linearly the apparent range one second earlier and one second later. Gates 0.98μsec wide are set at the extrapolated range position. Any three-second-old replies found within the earlier gate and zero-second-old replies found in the later gate are associated with the initial reply pair (formed from the two-second and one-second-old replies) as possible candidates for a track initiation. This process is repeated once every second so that any given reply could appear in two or more quadruplets that are formed as potential track initiations.

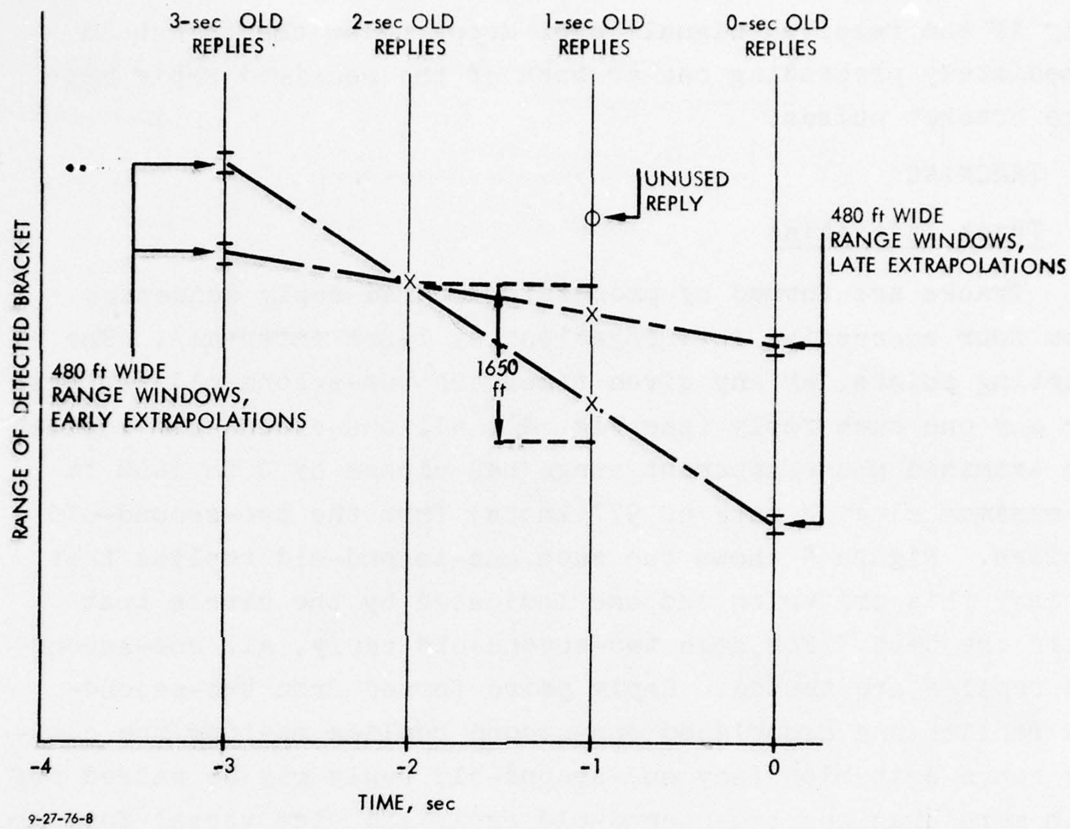


FIGURE 4. Range and Time of Replies Involved in Track Initiation

(This example shows two 1-sec old replies satisfying the closing range criterion with respect to one 2-sec old reply).

The binary sequences detected within each of the four brackets are logically "anded," and if the result is a legitimate altitude code, then a track is initiated. Subsequent track updates are obtained from received replies which meet the reply acceptance criteria implemented in the tracker continuation logic.

2. Track Continuation

After track acquisition, data must be collected for track continuation. The track is updated if and only if three conditions are satisfied.

First, at least one of the three C-bits in the altitude code between the two bracket pulses must be a "one." The second condition is an altitude correlation test. As the data are collected the altitude code of the reply message (to be referred to as the reply code) is correlated with a predicted altitude code which is derived from previous data. Actually, three codes are formed for use in the correlation process to allow for small changes in the intruder's altitude between data samples. The third condition requires that at least one "one" in the three C-bits of the apparent reply altitude code must agree with at least one "one" of the C-bits in one of the three predicted altitude codes.

The correlation procedure is as follows. Based on previous data, the predicted altitude of the intruder is computed. This estimate is rounded to the nearest 100 ft and a predicted altitude code z_p, is formed. Similarly, the altitude codes for z_p -100 ft and z_p + 100 ft are formed. The reply code is correlated with each of these and the one which produces the highest correlation value is labeled the "best." The correlation value, C, is calculated from the following equation:

$$C = CMAX - 3M - Q$$

where CMAX is a constant currently set to 48, M is the total

number of garbled 1"s in the reply code and Q is the total number of dropped 1"s in the reply code. Thus, if the reply code correlates perfectly with the predicted code, a correlation value of 48 is obtained. If there is less than perfect correlation, penalties are assigned both for 1's added by garble and for 1's dropped in the reply code. The penalty for the former is three times greater than for the latter.

The "best" code is used to update the altitude track of the intruder provided the correlation number exceeds some minimum value. The minimum value is expected to be set somewhere between 36 and 44. The data are used to update the tracks only if the correlation between the "best" code and the reply code exceeds the minimum value. If none of the reply messages exceed the minimum correlation value the track is not updated for that particular epoch. A track does not have to be updated for up to six consecutive epochs before it is dropped from the track file.

The MITRE BCAS employs a computer algorithm called the α-β tracker to maintain range tracks after acquisition. On the basis of range measurements obtained from reply signal time delays, the α-β tracker estimates range and range rates and predicts these for the subsequent interrogation-reply sequence. Tracker inputs are range measurements obtained from detected brackets of successive replies. The tracker equations are as follows:

$$\hat{r}_n = r_{np} + \alpha(r_n - r_{np}) \, ,$$

$$\hat{\dot{r}}_n = \dot{r}_{np} + \frac{\beta}{T}(r_n - r_{np}) ,$$

$$r_{np} = \hat{r}_{n-1} + T\,\hat{\dot{r}}_{n-1} \, ,$$

$$\dot{r}_{np} = \hat{\dot{r}}_{n-1} \, ,$$

where $\hat{r}_n$ and $\hat{\dot{r}}_n$ are estimated range and range rate for the nth sample after the range measurement r_n is obtained; r_{np} and $\dot{r}_{np}$ are the predicted range and range rate for the nth sample before the range measurement r_n is obtained; T is the time between samples (= 1 sec for MCAS); α and β are dimensionless tracker parameters.

An initiated and closing track* will be continued as long as the number of accumulated replies, or hits, exceeds a specified minimum which is a non-decreasing function of the number of interrogations. The minimum number of accumulated replies is given as a function of the number of interrogations by Table 1. A track which has been maintained for 30 interrogations (4 for initiation plus 26 for subsequent track extensions) is declared an established track.

F. THREAT LOGIC

An alarm will be generated by an established track if the projected time to a potential collision, for a BCAS-to-ATCRBS encounter, is approximately 30 sec or less. The logic used for an alarm is a modified form of that given in ANTC-117 (Ref. 1).

ANTC-117 provides a two-step threat algorithm; the first step, defined by a so-called modified tau condition, results in a warning which requires both encountering aircraft to stop turning; the second, defined by a modified tau condition with different parameters than the first, results in an alarm which requires both aircraft to perform complementary vertical escape

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*The ANTC-117 threat logic used with DABS transponders and BCAS-equipped aircraft allows for low range-rate opening close-in threats, while the logic for the ATCRBS mode excludes opening tracks. This is a possible inconsistency since such threats may be even more critically dangerous with that logic approach. The appropriateness of the MITRE logic for ATCRBS transponders (non-responding threats) is treated in IDA paper P-1215, published concurrently with this study.

TABLE 1. MINIMUM NUMBER OF ACCUMULATED REPLIES, L(n), NEEDED TO CONTINUE TRACK FOR n INTERROGATIONS AFTER ACQUISITION

Interrogation Number, n	Minimum Accumulated Replies L(n)
1	0
2	0
3	0
4	0
5	0
6	0
7	1
8	1
9	1
10	2
11	2
12	3
13	3
14	4
15	5
16	6
17	6
18	7
19	8
20	8
21	9
22	10
23	11
24	11
25	12
26	13

maneuvers. The logic of ANTC-117 provides relatively safe alarm/warnings for encounters between two CAS-equipped aircraft, both of which respond to the commanded maneuvers. In BCAS, MITRE has provided for the use of a DABS interrogator to furnish essentially interference-free communications with other BCAS- or DABS-equipped aircraft. Once an aircraft is identified as having BCAS or DABS the DABS mode of interrogations is used along with the alarm/warning logic of ANTC-117.

For the case of intruding aircraft which are equipped only with ATCRBS transponders, which do not receive alarm/warnings and which, therefore, will not initiate an avoidance maneuver, MITRE has provided a so-called "remitter" logic to be used when only one (the BCAS-equipped) aircraft is expected to perform an avoidance maneuver. The threat criterion for this logic is a single modified tau condition:

$$R \leq -\tau\dot{R} + R_o$$

in which

R = relative range
$\dot{R}$ = relative range rate (negative for closing)
R_o = constant equal to 1.0 nmi
τ = constant equal to 30 sec

The form of this "remitter" logic has been demonstrated to be proper for a CAS that can measure only range and range rate, but the selection of the constants is not proper for complete protection. In order to account for a possible acceleration of 1/2g by the intruder, not to mention measurement errors, R_o should be larger than one nmi. Also, the value of τ should include an allowance for the maximum warning delay expected as well as the pilot-aircraft reaction time and the time needed for maneuvering to safety, all of which add up to somewhat more than 30 sec.

G. DEGARBLING TECHNIQUES

MITRE plans to use one or both of two degarbling techniques in MCAS in order to reduce the amount of interference (garble) of a reply signal from an intruder by replies from other intruders. These techniques make use of inadvertent sensitivity and suppression recovery time variations in the transponder population to selectively interrogate transponders within limited sensitivity and suppression recovery intervals.

One, called "whisper-shout," is based on the variation of receiver sensitivity and other factors which limit the number of transponders affected by an interrogation at a particular signal strength. An interrogation is first transmitted at a very low power level. Next, a suppression pulse set (P_1, P_2) is transmitted at the same power level. This suppresses all transponders which have replied to the first transmission, so that a new interrogation at a higher power level will elicit replies from a new group of aircraft. Successive repetition of this procedure will divide the aircraft into different reply groups.

The whisper-shout technique is supposed to use four interrogation subsets based on four different MCAS transmission power levels. As originally planned*, the four power levels were 5.3 watts, 19.3 watts, 89.9 watts and 1000 watts.

Another technique for reducing garble that has been suggested by Lincoln Laboratories is called resuppression. Lincoln Laboratories' resuppression scheme is similar to the whisper-shout in that a sequence of interleaved suppression interrogation pulse pairs and mode-C interrogation pulse pairs are transmitted by BCAS. The separation between the suppression pair and the mode-C pair are varied to selectively interrogate the different groups of transponders. The operation

*Minor changes in these transmit power levels have recently been made and the new values are discussed in Appendix F.

depends on the tolerances that exist in the recovery of the transponders to a suppression pair. The specification on recovery is 25-45μs. Thus, if the first suppression pair and interrogation pair were separated by 30μs, only those transponders whose recovery times were less than 30μs would respond to the following mode-C interrogation pair. For the second group, two suppression pairs separated by 30μs are transmitted. These two suppression pairs effectively suppress the first group which is suppressed by each of the suppressions. The two suppression pairs are followed by a mode-C interrogation pair after a delay of about 5μs. Only those transponders which recover after the first pulse of the second suppression pair and before the first pulse of the mode-C interrogation pair reply in this group.

Subsequent groups are similarly stimulated to respond using two suppression interrogation pairs followed by mode-C interrogation pairs. Thus the population is divided into groups by virtue of the recovery times following a suppression interrogation pair.

Originally, for degarbling MITRE proposed using a four-quadrant sectored top antenna and an omni bottom antenna for MCAS using the whisper-shout technique. In high-density environments an MCAS using four power levels would transmit a sequence of four mode-C interrogations interleaved with three suppression interrogations in each quadrant of the top antenna and one sequence for the bottom antenna, for a total of 20 mode-C interrogations and 15 suppression interrogations. With the sectored antenna it was assumed that garble in a given range cell could be suppressed by at least a factor of 1/2 and that the four power levels would provide an additional factor of 1/2 - 1/4, for an effective total reduction of garble in a range cell of 1/4 - 1/8. If such reductions were deemed inadequate for high density traffic, it was suggested that the number of power levels be increased. Because of the general dissatisfaction with the garble reduction potential of the sectored

antenna and the perceived difficulty in installation of such an antenna, it has been abandoned and the Lincoln Laboratory resuppression scheme has been substituted.

Although the practicality of these garble reduction techniques still remains an open question, some informal projections* suggest that the transponder population might be sequentially interrogated in 16 nonoverlapping groupings (four whisper-shout groupings times four resuppression groupings). Appendix F casts some doubt on the efficacy of such techniques, as presently conceived, to overcome predicted garble levels.

*A rough draft for an Engineering Specification for BCAS.

IV. FACTORS WHICH DEGRADE MCAS

A. INTERROGATION RATES

In the worst case of interference and transponder blockage the estimated inability of a transponder to respond to an interrogation waveform should be that for an aircraft in the center of the 1982 Los Angeles Traffic Model. In making this estimate it will be assumed that the aircraft is equipped with an ATCRBS transponder having a receiver MDS of -71 dbm and an idealized omnidirectional antenna with 0 db gain.

Such an aircraft will respond to airborne MCAS interrogations within 3.2 nmi during the first whisper-shout interrogation subset (Cf. Section III-G) and will be suppressed by these interrogators during interrogation subsets two through four. The aircraft will respond to airborne MCAS interrogators located between 3.4 nmi and 6.4 nmi from the center of the traffic model during interrogation subset two and will be suppressed by these aircraft during subsets three and four. The aircraft will receive one mode C interrogation and one SLS waveform from airborne interrogators located between 6.4 nmi and 13.7 nmi from the center of the traffic model and will receive only a mode C interrogation from the MCAS aircraft located between 13.7 nmi and 46 nmi from the center of the traffic model.

Based on the 1982 Los Angeles traffic model and assuming 10 percent of the aircraft have MCAS transponders, the expected number of MCAS interrogators in each of the above-mentioned range intervals is 2, 6, 13, and 52, respectively. This results in the aircraft at the center of the traffic model receiving, on the average, 73 mode C interrogations per second and 31 SLS waveforms per second.

The ground-based interrogation rate will be estimated here by assuming that there are 20 SSRs within communication range of an aircraft at the center of the Los Angeles Basin Traffic model. An SSR radar has an interrogation rate of about 400 pulses per second, a main beam width of about 3.6° (one percent of 360°) and an antenna rotation rate of about once every 5 seconds.* During each antenna scan the radar will interrogate an aircraft about 20 times during a single rotation or at an average rate of 4 per second. The combined rate for 20 SSRs is 80 per second.

When the radar's antenna sidelobes are pointed at the aircraft the communication range of the radar is greatly reduced and it is unlikely that the received signal will exceed the MDS threshold of the airborne transponder. Should the interrogation waveform cross the receiver threshold when the sidelobes illuminate the aircraft, the SLS waveform will keep the transponder from responding. The SLS waveform is transmitted from an omnidirectional antenna on the ground and will be received by the aircraft at a higher level than the radar's interrogation waveform unless the antenna main lobe is pointing in the aircraft's direction.

The gain in the antenna sidelobes region is much less than the main beam so that the radar's communication range is much less when the sidelobes are pointing at the aircraft. The analysis will therefore assume that only two of the SSR radars are close enough so that the transmitted SLS waveform exceeds the airborne receiver's MDS threshold.

The two radars that are assumed to be close enough to allow their SLS waveforms to exceed the airborne receiver's

*There are two kinds of SSR radars. The long-range radars have a rotation rate of about 0.1 RPS and the short-range radars have a rotation rate of about 0.2 RPS.

threshold will cause the airborne transponder to be suppressed at a rate of about 792 times per second.*

B. TRANSPONDER BLOCKAGE

A transponder may not always be able to reply to an MCAS interrogation, because it is already occupied with processing or replying to other interrogators. This kind of failure to reply is known as transponder blockage. There are other sources of reply failure, such as equipment malfunction and excessive path losses, particularly from severe antenna nulls, but these effects are ordinarily not considered part of blockage.

After each interrogation of an ATCRBS transponder some dead time ensues during which other interrogations are blocked. The ATCRBS standard (Appendix A) specifies that the dead time associated with the reception of an interrogation waveform shall be no more than 125 μsec after the transmission of the last reply pulse (i.e., an F_2 framing pulse). The F_2 pulse is transmitted 20.3 μsec after the F_1 pulse and the delay between the reception of the interrogation waveform and the transmission of the F_1 pulse cannot be more than 3.5 μsec. The maximum transponder dead time is therefore 148.8 μsec.

A transponder's suppression time is associated with the reception of an SLS waveform (i.e., P_1 P_2 pulse pair). The (SLS) waveform prevents a transponder from replying to an interrogation waveform and is used primarily to prevent a response when an aircraft is being illuminated by the sidelobes of the ground-based radars. The ATCRBS specification for suppression time is 35 μsec ± 10 μsec so that a maximum of 45 μsec is possible.

*Radars have an interrogation rate of about 400 waveforms per sec and about one percent of these are transmitted within the main beam. As a result an airborne transponder will be suppressed about 792 times per sec by two nearby radars.

The magnitude of blockage is a function only of the interrogation rate from all interrogators, SSRs and MCAS, within communications range of a given transponder. Due to blockage alone, the reduction in the probability of transponder reply in dense traffic is estimated at between 0.90 and 0.95.

C. GARBLE

Garble is the interference experienced by a reply signal when it is overlapped by another reply. When this occurs the altitude code between the framing pulses (F_1 and F_2) of a reply signal can be altered by the addition or destruction of bits. In fact, garble can cause the loss of a reply signal altogether by interfering with the detection of its framing pulses.

The garble rate γ is the average number of replies per sec received by an MCAS. Since the length of a reply signal is 20.3 μsec an equivalent parameter $N = 20.3\ \gamma$ is used in this report and referred to as the number of overlapping replies. MITRE has defined the number of overlapping replies to be the number which overlap a given reply by arriving no earlier and no later than 20.3 μsec. The number of overlapping replies defined by MITRE is equal to twice the number N used here.

An MCAS can experience two types of garble: synchronous, due to replies stimulated by the MCAS, itself, and asynchronous, due to replies stimulated by other MCAS or by ground-based SSRs which are part of the ATCRBS. Most of the interference affecting MCAS performance will be due to synchronous garble, although asynchronous garble will generally contribute a nonnegligible part of the number of overlapping replies relative to the MCAS.

Some additional garble will occur because of multipath effects which are discussed in Appendix D. However, this report does not include the multipath contribution in its quantitative analysis of MCAS performance.

D. GARBLE RATES

A discussion and analysis of the garble level expected in the 1982 Los Angeles Basin is presented in Appendix F. A plot of the aircraft density in the 1982 Los Angeles Basin is presented in Fig. 5. In the figure, the number of aircraft within a 1.644 nmi range interval is plotted as a function of range from the center of the static traffic model. A range interval of 1.644 nmi is of interest in that it corresponds to a two-way transmission delay of 20.3μsec. A reply message has a time duration of 20.3μsec so that the number of overlapping reply messages is determined by the number of aircraft within the 1.644 nmi range interval which respond to an interrogation.

The data presented in Fig. 5 show a maximum of 41 aircraft within a 1.644 nmi range interval. Although at the maximum instrumented range of the MCAS the number of aircraft within this range interval drops to 35, this still represents a severe environment in which to operate. In it the MCAS will not be able to perform its functions satisfactorily unless effective garble reduction techniques are devised and implemented in the system. Appendix F assesses the compatibility of the whisper-shout and resuppression garble reduction described in Section III-G. It is not certain that these techniques are compatible; however, if they are, it may be possible to reduce the maximum number of aircraft that will reply within a 20.3 μsec period to about 6.6 (cf. Appendix F). This represents an effective reduction in the synchronous garble level by a factor of about 1/6 over that which would be indicated for the peak aircraft density presented in Fig. 5.

Synchronous garble is a direct result of the interrogations transmitted by the threatened MCAS equipped aircraft. Aircraft replying to these interrogations are close enough together so that their reply messages overlap. Based on the estimate provided in Appendix F, an aircraft in the center of the 1982

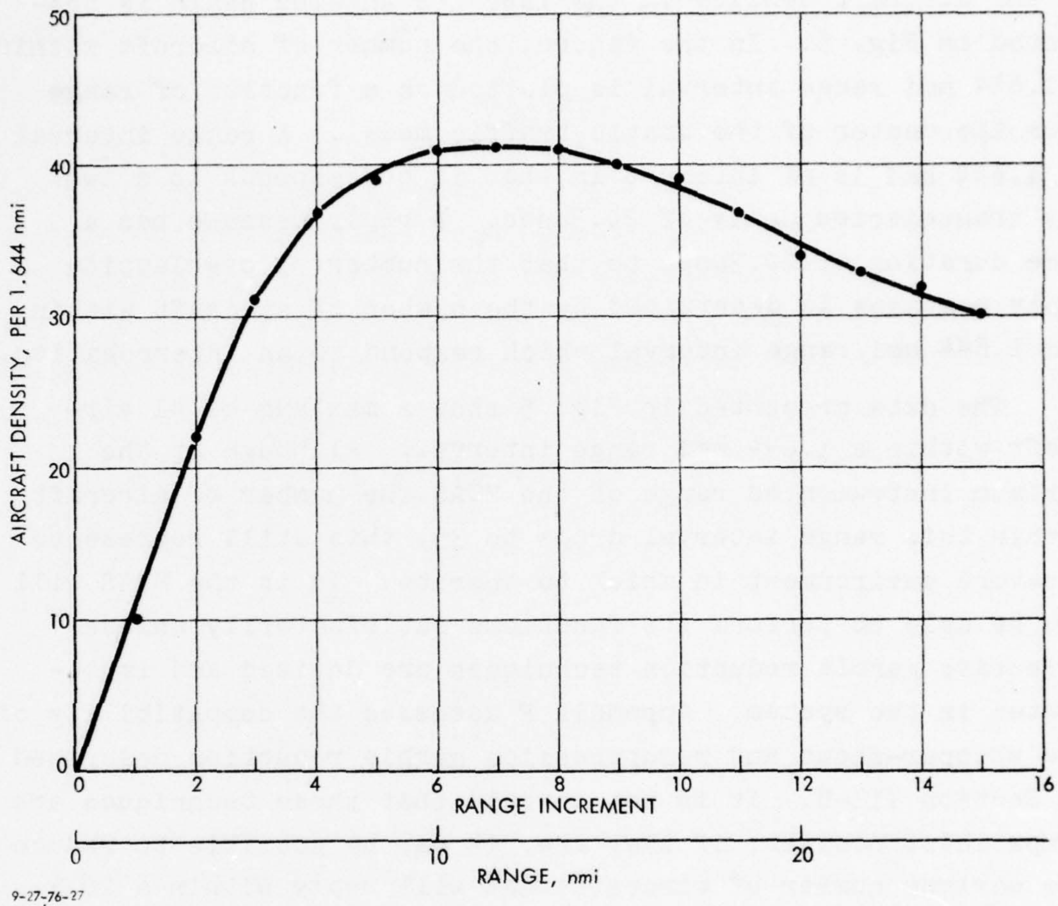

FIGURE 5. Aircraft Density in 1982 Los Angeles Basin Within a 1.644 nmi Range Segment

Los Angeles Basin can expect 6.3 overlapping replies due to synchronous garble generated at a range of 20 nmi. (This assumes that the whisper-shout and resuppression garble reduction techniques are compatible). Garble or overlapping reply messages will also result when the aircraft, in the vicinity of the threatened MCAS-equipped aircraft, reply to ground radar interrogations and to interrogations from other MCAS-equipped aircraft.

We have assumed 20 SSR radars in the Los Angeles Basin. Each radar will interrogate each aircraft within its communication range at a rate of about 4 interrogations per second. Not all of the aircraft will be within communication range of the 20 radars; however, if they were, about 64,000 reply messages would be generated each second. This corresponds to an average of 1.2 reply messages in 20.3 μsec. In the analysis, however, it will be assumed that an MCAS in the center of the Los Angeles Basin receives only one quarter of this number of radar induced reply messages (i.e., an average of 0.3 per 20.3 μsec).

Interrogations from other MCAS-equipped aircraft will create garble for an aircraft in the center of the Los Angeles Basin. The communication range between aircraft is such that an interrogation which originates near the center of the Basin will cause practically all of the aircraft to reply. An interrogation which is transmitted near the edge of the traffic model will cause the aircraft near the center to reply but not the aircraft on the far side of the model as viewed from the interrogating aircraft.

If all of the MCAS interrogators (assumed to be 10 percent of the population) were concentrated in the center, the reply rate due to airborne interrogations would be about 64,000 reply messages per second or about 1.3 reply messages per 20.3 μsec. If all of the airborne interrogations were along the edge of the traffic model, the reply rate due to other MCAS-

equipped aircraft would be nearly cut in half (i.e., to about 32,000 replies per second or 0.65 replies per 20.3 μsec). The traffic density is highest near the center so that the reply rate due to airborne MCAS interrogations should be closer to 1.3 than to 0.65 replies per 20.3 μsec. For estimating the system's probability of warning, it will be assumed that the reply rate is half way between these extremes (i.e., about 1.0 replies per 20.3 μsec).

The estimates of garble over the Los Angeles Basin in 1982 can be summarized as follows. An MCAS-equipped aircraft at the center of the 1982 Los Angeles Basin will experience an average synchronous garble level of 6.3 overlapping replies due to its own interrogations. This estimate assumes the implementation of the whisper-shout and the resuppression garble reduction techniques. In addition, ground radars and fruit due to other MCAS interrogators will contribute to the garble level by creating 1.3 overlapping replies in the form of asynchronous garble.

V. MCAS WARNING CAPABILITY

A. TIMING REQUIREMENT

Since MCAS requires 26 interrogations after an initial target acquisition (four successive target replies) before a track is considered established and threat evaluation can begin, it has a relatively low inherent effective data rate. Track establishment depends upon a minimum number of target replies as a function of the number of interrogations in order to maintain a track file. The file is updated every second so that the system can make continual use of reply data already obtained. MCAS is supposed to track all intruder aircraft in this way within a range of 20 nmi and provide protection (i.e., alarms) against any encounter in which the closing rate is no greater than 1650 ft/sec.

The MCAS system concept is based upon the idea that any intruder will enter the surveillance region at least 30 seconds before it reaches the alarm boundary as well as the implied requirement that the system be able to establish a track on every such intruder in time to perform a threat evaluation and an escape maneuver when necessary. Assuming that MCAS has the capacity to maintain track on all intruders, the system concept has two possible flaws.

First, an intruder may enter the surveillance region too close in range, e.g., by taking off from a nearby airstrip, becoming a "pop-up target." MITRE has recognized this difficulty but has not yet proposed a complete solution. Only for the condition that a track is acquired without interference from garble has MITRE proposed to abbreviate the track establishment procedure. Therefore, in the analysis and evaluation

of MCAS no attempt has been made to include in this report the possibility of such an event with garble conditions representative of heavy traffic.

Second, because of transponder blockage and garble, the establishment of an intruder track may be delayed too long to permit adequate warning time. The probability of such an occurrence will be estimated in Subsection B which follows.

Adequate warning time can be defined in terms of the specified maximum closing rate of 1650 ft/sec and the maximum surveillance range of about 20 nmi. For the "remitter" logic a 1650 ft/sec closing rate should result in a warning when the range between the encountering aircraft drops to 9.4 nmi. (This assumes a tau slope of 31 sec, to account for a 1-epoch processing time for threat evaluation, plus an offset range R_o = 1 nmi). The difference between the maximum instrumented range and the warning range of 9.4 nmi is 10.6 nmi. The aircraft will traverse the distance in 39 sec at the assumed value of range rate so that the acquisition and required tracking must be completed in 39 epochs. To fulfill this requirement a maximum delay in acquisition of up to 9 epochs or 9 sec is permitted.

A similar estimate of the allowable delay in track acquisition can be made in the same manner for any other closing rate. A curve for the remitter logic (31, 1.0) showing the allowable delay as a function of the closing rate is given in Fig. 6. Also included for comparison are similar curves corresponding to the two-tau warning and alarm criteria of ANTC-117 which would be used if both encountering aircraft were equipped with MCAS or DABS. Note that the τ_2 curve indicates an even smaller allowable delay than is indicated by the single tau curve. In what follows, the discussion will be concerned primarily with the remitter logic rather than the ANTC-117 logics for several reasons. Initially, it is not expected that

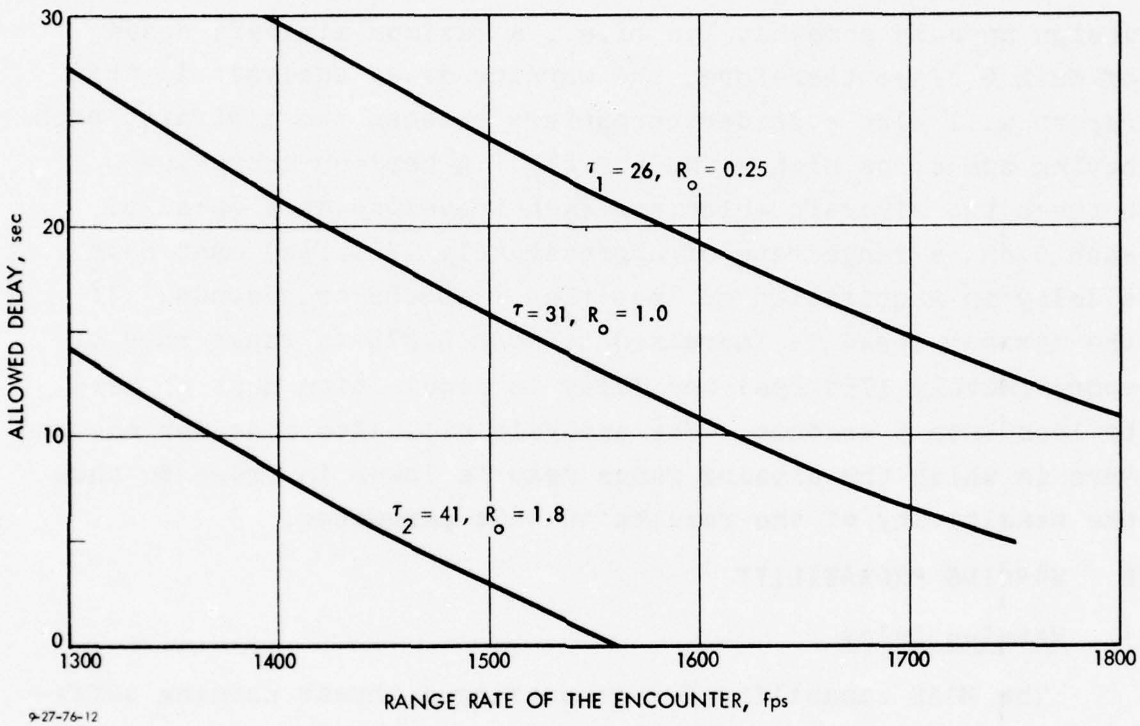

FIGURE 6. Allowed Acquisition Delay

all aircraft qualified to carry MCAS or DABS will be so equipped, so that the remitter logic is appropriate. If, as presently is the intention, the FAA adopts a policy to make MCAS optional, many high performance general aviation and military aircraft may not be equipped with MCAS and may not be equipped with DABS, so that the remitter logic may still be appropriate.

The maximum range rate of the currently specified MITRE design appears somewhat low (i.e., a maximum aircraft speed of Mach 0.825); therefore, the warning delay analysis in this report will also consider encounters between two aircraft, each having speeds as high as Mach 0.875. A head-on encounter between two aircraft which are each traveling at a speed of Mach 0.85 (a range rate of approximately 1700 fps) must have a delay in acquisition of less than 7 epochs or seconds. If the maximum speed is increased to Mach 0.875 (a range rate of approximately 1750 fps) the delay in acquisition must be held to less than 5 seconds. The analysis will also consider encounters in which the closing range rate is lower in order to show the sensitivity of the results to this parameter.

B. WARNING PROBABILITY

1. Warning Delay

The MCAS capability for generating a threat warning sufficiently early to permit an avoidance maneuver depends upon the probability that a warning will occur within a given time delay. That delay is a function (Fig. 6) of the closing rate between the encountering aircraft. The probability that the warning will occur soon enough depends upon the probabilities that within a specified time: (1) a reply is received and detected (the round reliability); (2) a track is acquired; (3) the track is established.

2. Round Reliability

The term "round reliability" will be used to denote the probability that the MCAS interrogating aircraft receives and

detects the bracket pulses associated with the reply message from the threatening aircraft. The threatening aircraft must first receive the MCAS interrogation and transmit its reply message. The reply message must then be received with sufficient amplitude to exceed the MDS threshold of the receiver in the MCAS-equipped aircraft.

The probability of completing the process up to this point will be referred to as the system's reply probability. After this operation, the bracket pulse pair must be detected before the interrogation/reply sequence is successfully completed. The likelihood of successfully completing the latter operation will be referred to as the bracket pulse detection probability.

A transponder will not reply to an interrogation if it has previously received an interrogation or suppression from another interrogator and is occupied in, or has not fully recovered from, processing the latter signal. This inability of the transponder to reply to an interrogation is called blockage.

The effect of transponder blockage can be estimated by assuming that the interrogation and suppression waveforms occur randomly in time and that blockage statistics follow a Poisson distribution. Under these conditions the reply probability is given by

$$P_r = e^{-(f_{ai}\,\tau_i + f_{as}\,\tau_s)}\, e^{-(f_{gi}\,\tau_i + f_{gs}\,\tau_s)} \qquad (1)$$

where f_{ai} is the airborne interrogation rate, τ_i is the transponder blockage time associated with the reception of an interrogation waveform, f_{as} is the airborne SLS waveform transmission rate, τ_s is transponder suppression time, and f_{gi} and f_{gs} are the interrogation and SLS waveform transmission rates from ground radars. The first exponential give the reply probability due to airborne interrogators and the second gives the reply

probability due to ground-based radars. Substituting into equation (1) the estimates for the parameters derived in Sections IV-A and IV-B gives

$$P_r = e^{-[(73)(148.8) + 31(45)]x10^{-6}} \; e^{-[80(148.8)+792(45)]x10^{-6}}$$

$$= e^{-0.012} \; e^{-.047} = 0.94$$

Thus, the assumed airborne MCAS population reduces round reliability by only one percent relative to that caused by SSRs alone.

The above estimate of reply probability is approximately equal to more optimistic estimate (0.95) for current high density traffic regions. The latter value (0.95) has been used to calculate the expected delay in acquiring a threatening aircraft. Such factors as equipment failures and antenna nulls have been neglected. These effects are difficult to estimate and will not be included in the analysis. Also, the analysis assumes that only the whisper-shout garble reduction technique is incorporated in the MCAS design. If the resuppression technique is also used, transponder blockage will increase and the reply probability will decrease.

The probability of transponder reply has been evaluated for a less severe environment in Ref. 3. A 150-aircraft deployment, within a radius of 200 nmi and centered on JFK Airport, New York, was used in that analysis. In all, 64 aircraft were MCAS-equipped so that the total number of airborne interrogations for the analysis of Ref. 3 was 80 percent of that considered in the analysis presented here for the 1982 L.A. Basin. Reference 3 also included between 225 and 292 ATCRBS ground interrogators; however, it does not specify the average rate at which an airborne transponder was interrogated from the ground.

The results indicate that the probability of transponder reply will change very little with MCAS. With 225 ground

interrogators, assuming that all aircraft are equipped with an air carrier type of transponder, the transponder reply probability was 0.95 with and without MCAS. If all aircraft were equipped with general aviation transponders the reply probability was 0.98 with and without MCAS. For all aircraft equipped with military transponders the reply probability was 0.94 without MCAS and 0.93 with MCAS. The estimate of the transponder reply probability given here seems to be in general agreement with the estimates of Ref. 3.

The detection of the bracket pulses associated with a reply message requires that the signal level drop below the MDS threshold immediately preceeding at least one of the bracket pulses. In a garbled environment this cannot be assured, and the bracket pulse detection probability will therefore be less than unity even if the reply message signal level is adequate for detection.

There are two ways that garble can prevent bracket pulse detection of the threatening aircraft reply message. An F_1 framing pulse associated with garble can occur immediately before the threatening aircraft's F_1 framing pulse. (For ease of reference, these two framing pulses will be denoted as F_{1G} and F_{1T}, respectively). Framing pulses are always 20.3 nsec apart so that an F_2 pulse due to garble (i.e., F_{2G}) would also appear immediately before the threatening aircraft's F_2 framing pulse (i.e., F_{2T}). The second way for garble to interfere with the bracket pulse detection is for any type of garble pulse, other than a F_{1G}, to appear immediately before F_{1T} and for any type of garble pulse, other than a type F_{2G}, to appear immediately before F_{2T}. (A type F_{2G} pulse occurs before F_{2T} if, and only if, an F_{1G} preceeds F_{1T}. This possibility is included in the first way that bracket detection is prevented by garble).

The timing between a garble pulse and a framing pulse is critical in determining whether a bracket pulse detection is

missed. This timing is discussed as if only one garble pulse and one framing pulse were involved. However, it should be remembered that garble must interfere with both of the threatening aircraft's framing pulses in order to prevent the generation of an appropriate bracket pulse detection signal.

Interference to the detection of bracket pulses occurs when the trailing edge of a garble pulse is within 121 nsec (i.e., one clock count) of the leading edge of the threatening aircraft's bracket pulse. In other words, the garble pulse and the bracket pulse must be separated (as measured from leading edge to leading edge) by less than 570 nsec. The interference will continue as the separation between the pulses decreases until finally the threatening aircraft bracket pulse and the garble pulse completely overlap.

When this happens it is possible for the phase difference between the two pulses to cause them to add destructively, and if the amplitudes are about equal the resulting signal can drop below the MDS threshold. It is unlikely for the latter combination of conditions to occur, and, in general, a usable bracket pulse detection signal will result when the threatening aircraft's bracket pulse and a garble pulse completely overlap.

Actually, a useful bracket pulse detection signal can be expected when the garble pulse preceeds the threatening aircraft bracket pulse by a small amount. For example, the acquisition process would not be encumbered if, due to garble, the bracket pulse detection signal occurs one clock count earlier than it would without garble.

A one clock count timing error in the range and altitude code read command will not cause a loss of data. A two clock count error is more serious since this will cause the equipment to check for the presence of the altitude code pulses long before the peak amplitude of the pulses occurs.

In addition, the ATCRBS specification allows a pulse interval tolerance of 100 nsec so that a two clock count error in the read command can result in the equipment attempting to monitor the presence of an altitude pulse 350 nsec before the center of the pulse occurs in time. The time duration of the pulses is only 450 nsec; thus, it is likely that an altitude pulse will be missed if the read command is two clock counts early.

If a bracket pulse detection signal occurs two clock counts early, it is possible and even likely that some of the pulses which are transmitted as part of the reply message altitude code will be missed and a "1" will be read as an "0". During the acquisition process the altitude codes from the four acquisition reply messages are "and" together to produce the predicted code. As a result, a missed altitude bit in any one of the four required reply messages will cause an extra "0" or missing "1" to appear in the predicted altitude code.

The effect of missing "1's" in the predicted altitude code is severe since in the correlation process (Cf. Section III-E-2) during track continuation after acquisition this error gives the appearance of garbled "1's" in the reply code. An apparent garbled "1" in the reply code is heavily weighted in computing the correlation process and results in a reduction of 3 in the correlation number. In fact, no more than two garbled "1's" can be tolerated if a minimum correlation value of 40 is to be obtained. However, it is likely that this minimum will be chosen to be less than 40 (Cf. Section VI-B-2) for other reasons.

It is also possible for garble during the track continuation reply message to produce garbled "1's" in the reply code. It is the combination of missing "1's" in the predicted code and garbled "1's" in the track continuation reply code which must be held to a maximum of two if the minimum correlation value is set to 40. This seems dubious if one of the bracket

pulse detection signals occurs two clock counts early during the acquisition process.

The forthcoming analysis will assume that a two clock count error in the occurrence of the bracket pulse detection signal cannot be tolerated so that the time during which garble pulses interfere with the bracket pulse detection of the threatening aircraft's reply extends from 180 nsec (i.e., 1-1/2 clock counts) to 570 nsec. This corresponds to a time window which is 390 nsec wide, during which a garble pulse causes interference. For ease of reference this time window will be called the interference window.

In the analysis, we need to distinguish between different types of garble pulses. Framing pulses, whether they be F_1 or F_2 occur at the same rate as reply messages while pulses associated with the altitude code of a garble reply message are more prevalent. Appendix E shows that, on the average, a mode C reply message will contain six pulses. There are always two framing pulses in a reply message; thus, the altitude code will, on the average, consist of four pulses. In terms of the reply message transmission rate, γ, the rate of occurrence of F_{1G} pulses is γ, the rate of occurrence of F_{2G} pulses is also γ, and the rate at which altitude code pulses occurs is, on the average, 4γ.

Garble pulses are assumed to be randomly dispersed in time and the statistics associated with their occurrence within an interference window is assumed to follow a Poisson distribution. If garble pulses occur at an average rate of m times per second and the time window width is τ, the probability that a pulse will not appear in the window is

$$P \text{ (no pulse)} = e^{-m\tau} .$$

The probability that at least one pulse will appear in the window is one minus this quantity.

As mentioned earlier, two sets of conditions will prevent the MCAS from generating a satisfactory bracket pulse detection signal. First of all, an F_{1G} garble pulse can appear in the interference window preceeding F_{1T}. The probability that this will occur is

$$P(I_1) = 1 - e^{-\gamma \times 390 \times 10^{-9}} \qquad (2)$$

The other set of conditions requires an F_{2G} or an altitude code pulse to appear in the interference window preceeding F_{1T} and for an F_{1G} or an altitude code pulse to appear in the interference window preceeding F_{2T}. The probability that this will occur is

$$P(I_2) = \left(1 - e^{-5\gamma \times 390 \times 10^{-9}}\right)^2 \qquad (3)$$

If the two conditions which cause interference for the detection of bracket pulses are independent, the probability of interference due to either is

$$P(I) = P(I_1) + P(I_2) - P(I_1, I_2)$$

$$= P(I_1) + P(I_2) - P(I_1)\,P(I_2) \qquad (4)$$

Equations (2), (3), and (4) have been evaluated and the result was subtracted from unity to give the MCAS bracket pulse detection probability. The results are presented in Table 2a as a function of the number of overlapping reply messages (N).

TABLE 2a. BRACKET PULSE DETECTION PROBABILITY

Number Overlapping Reply Messages (N)	Bracket Pulse Detection Probability
1	0.973
2	0.933
3	0.885
4	0.832
5	0.776
6	0.720
7	0.665
8	0.611
9	0.559
10	0.511
11	0.465
12	0.422
13	0.383
14	0.346
15	0.313
16	0.282

Round reliability is calculated by multiplying the bracket pulse detection probability of Table 2a by 0.95 (the estimated reply probability). These calculations are carried out in Table 2b.

TABLE 2b. ROUND RELIABILITY AS A FUNCTION OF GARBLE LEVEL

Number of Overlapping Reply Messages (N)	Bracket Pulse Detection Probability	Reply Probability	Round Reliability
0	1.0	0.95	0.95
1	0.973	0.95	0.924
2	0.933	0.95	0.886
3	0.885	0.95	0.841
4	0.832	0.95	0.790
5	0.776	0.95	0.737
6	0.720	0.95	0.684
7	0.665	0.95	0.631
8	0.611	0.95	0.580
9	0.559	0.95	0.531
10	0.511	0.95	0.485
11	0.465	0.95	0.442
12	0.422	0.95	0.401

3. Probability of Acquiring Enough Data For Warning

The next step in determining the probability of warning is to derive the relationship between round reliability and the probability of acquisition with less than a specified delay. This is accomplished in Appendix B and results are summarized in Fig. 7.

The garble level confronting an MCAS-equipped aircraft is assumed to have a Poisson distribution so that the occurrence of any particular garble level is to be associated with a probability which depends on the expected mean value. This subsection will only consider mean garble levels of up to 7 overlapping replies per 20.3 μsec. This is actually a higher garble level than MITRE anticipates being able to handle* and the results show that system performance at such a garble level is poor.

Sample calculations for determining the probability that the MCAS will obtain adequate tracking data (i.e., a 30-second track) before an intruder penetrates the threat boundary is shown in Table 3. These calculations are for an acquisition delay of 7 or less seconds and therefore correspond to an encounter with a range rate of 1700 fps.

The results presented in the table are as follows. The first column gives a particular garble level in which target acquisition is to be accomplished. The next two columns give the round reliability for each garble level and the probability of acquisition for each garble level with a delay of less than the specified amount. The following 14 columns are grouped in

*MITRE believes the performance of the MCAS range tracker will be satisfactory at a garble level of 8 overlapping replies in ± 20.3 μsec or 40.6 μsec. This is equivalent to only 4 overlapping replies in 20.3 μsec.

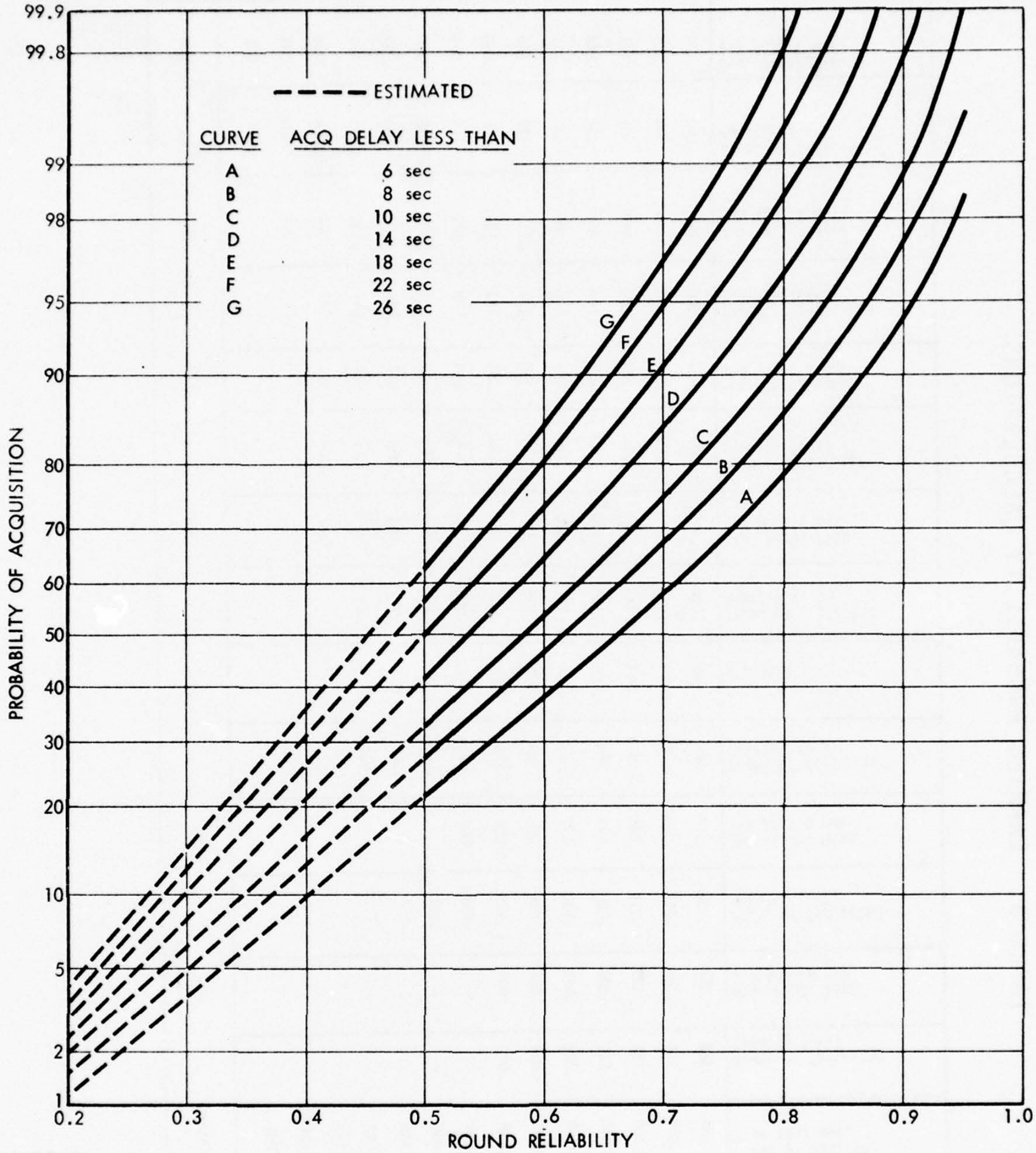

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FIGURE 7. Acquisition Probability as a Function of Round Reliability

TABLE 3. WARNING PROBABILITY CALCULATIONS

Number Overlapping Reply Messages (N)	Round Reliability	Prob. of Acq. in 7 or less sec.	Prob. of Indicated Overlap (y=1)	Acq. Prob. Dist. y=1 AC/1.6 nmi	Prob. of Indicated Overlap (y=2)	Acq. Prob. Dist. y=2 AC/1.6 nmi	Prob. of Indicated Overlap (y=3)	Acq. Prob. Dist. y=3 AC/1.6 nmi	Prob. of Indicated Overlap (y=4)	Acq. Prob. Dist. y=4 AC/1.6 nmi	Prob of Indicated Overlap (y=5)	Acq. Prob. Dist. y=5 AC/1.6 nmi	Prob. of Indicated Overlap (y=6)	Acq. Prob. Dist. y=6 AC/1.6 nmi	Prob. of Indicated Overlap (y=7)	Acq. Prob. Dist. y=7 AC/1.6 nmi
0	0.95	0.995	.368	.366	.135	.134	.050	.050	.018	.018	.007	.007	.002	.002	.001	.001
1	0.924	0.990	.368	.364	.268	.268	.149	.148	.073	.072	.034	.034	.015	.014	.006	.006
2	0.886	0.960	.184	.177	.271	.260	.224	.215	.147	.141	.084	.081	.045	.043	.022	.021
3	0.841	0.915	.061	.056	.180	.165	.224	.205	.195	.178	.140	.128	.069	.081	.052	.048
4	0.790	0.850	.015	.013	.090	.077	.168	.143	.195	.166	.175	.149	.134	.105	.091	.077
5	0.737	0.742	.003	.002	.036	.027	.101	.075	.156	.116	.175	.130	.161	.119	.128	.095
6	0.684	0.644	.001	.001	.012	.008	.050	.032	.104	.067	.146	.094	.161	.104	.149	.096
7	0.631	0.530	-	-	.003	.002	.022	.012	.059	.031	.104	.055	.138	.073	.149	.079
8	0.580	0.430	-	-	.001	-	.008	.003	.030	.013	.065	.028	.103	.044	.130	.056
9	0.531	0.330	-	-	-	-	.003	.001	.013	.004	.036	.012	.069	.023	.101	.033
10	0.485	0.245	-	-	-	-	.001	-	.005	.001	.018	.004	.041	.010	.071	.117
11	0.442	0.195									.008	.002	.023	.004	.045	.009
12	0.401	0.155											.011	.002	.026	.004
13	0.364	0.110													.014	.002
Cumulative Probability				.979		.941		.884		.807		.719		.624		.544

pairs and provide the calculations for determining the cumulative probability for mean garble levels between 1 and 7 overlapping replies. The first of each pair of columns gives the probability that the number of overlapping replies specified on the left is obtained. (This, of course, depends on the mean garble level). The second column of each pair is the product of the first column of each pair and the probability of acquisition listed in column 3. The sum of the second column of each pair is given at the bottom of the table. This value is the cumulative probability for each of the mean garble levels associated with the column pair.

A summary of the results of the calculations is presented in Fig. 8. Six curves are presented for encounters with different values of range rate between 1340 f/sec and 1750 f/sec. These data are plotted on probability paper; thus, the system performance drops off rapidly as the garble level increases. For example, for an encounter with a range rate of 1650 f/sec, the system has a probability of 0.98 of acquiring sufficient tracking data for warning if the average garble level is 1.3 overlapping replies in 20.3 μsec. At twice this garble level the probability drops to 0.93, at three times the garble level (3.9 overlapping replies per 20.3 μsec) the probability is 0.85, and at 4 times the garble level (5.2 overlapping replies per 20.3 μsec) the probability is 0.76.

The data of Fig. 8 have been replotted in a slightly different form in Fig. 9. The curves presented in this figure are for a given level of performance (i.e., a constant probability of obtaining sufficient tracking data for warning) and the variables are the garble level and range rate of the encounter. Several scales are included on the abscissa to allow the range rate of the encounter to be interpreted in terms of aircraft speed (each aircraft is assumed to be traveling at the same speed) and the crossing angle between

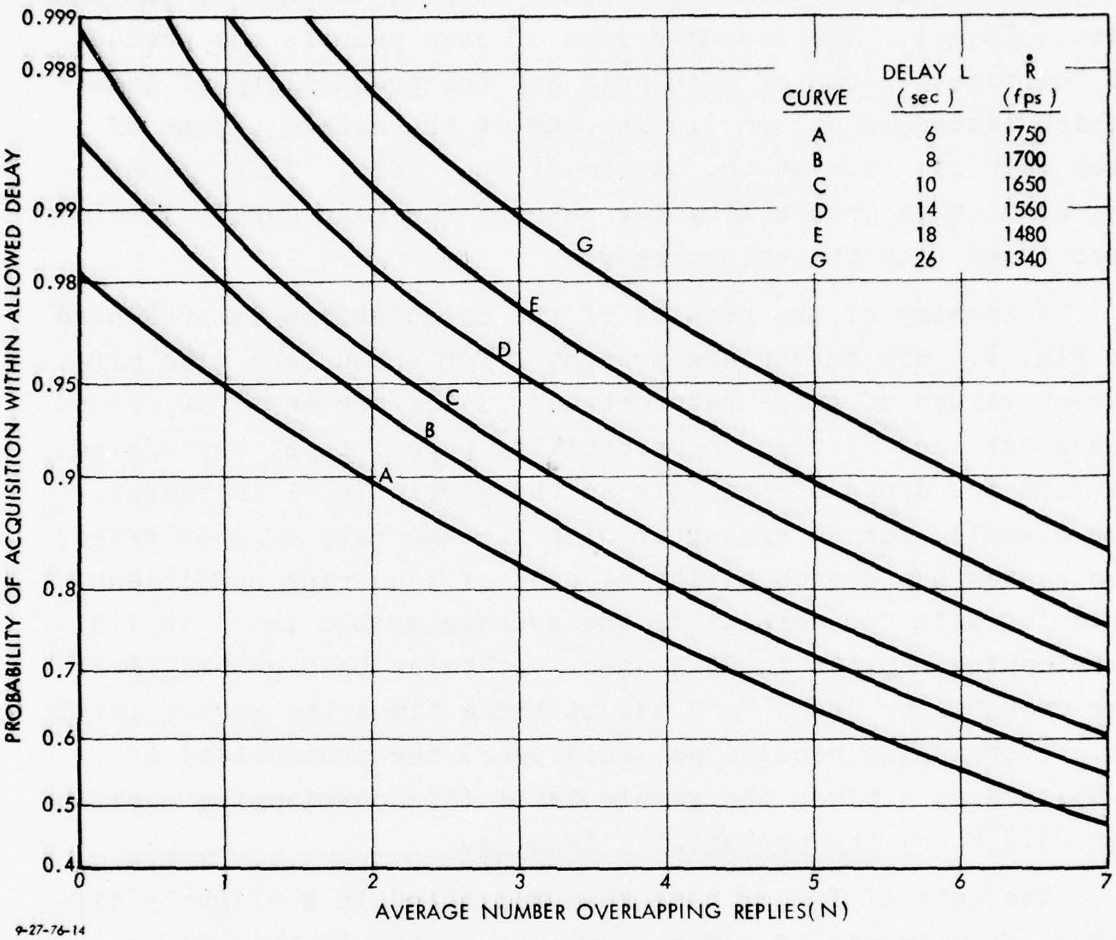


FIGURE 8. MCAS Probability of Obtaining Sufficient Tracking Data

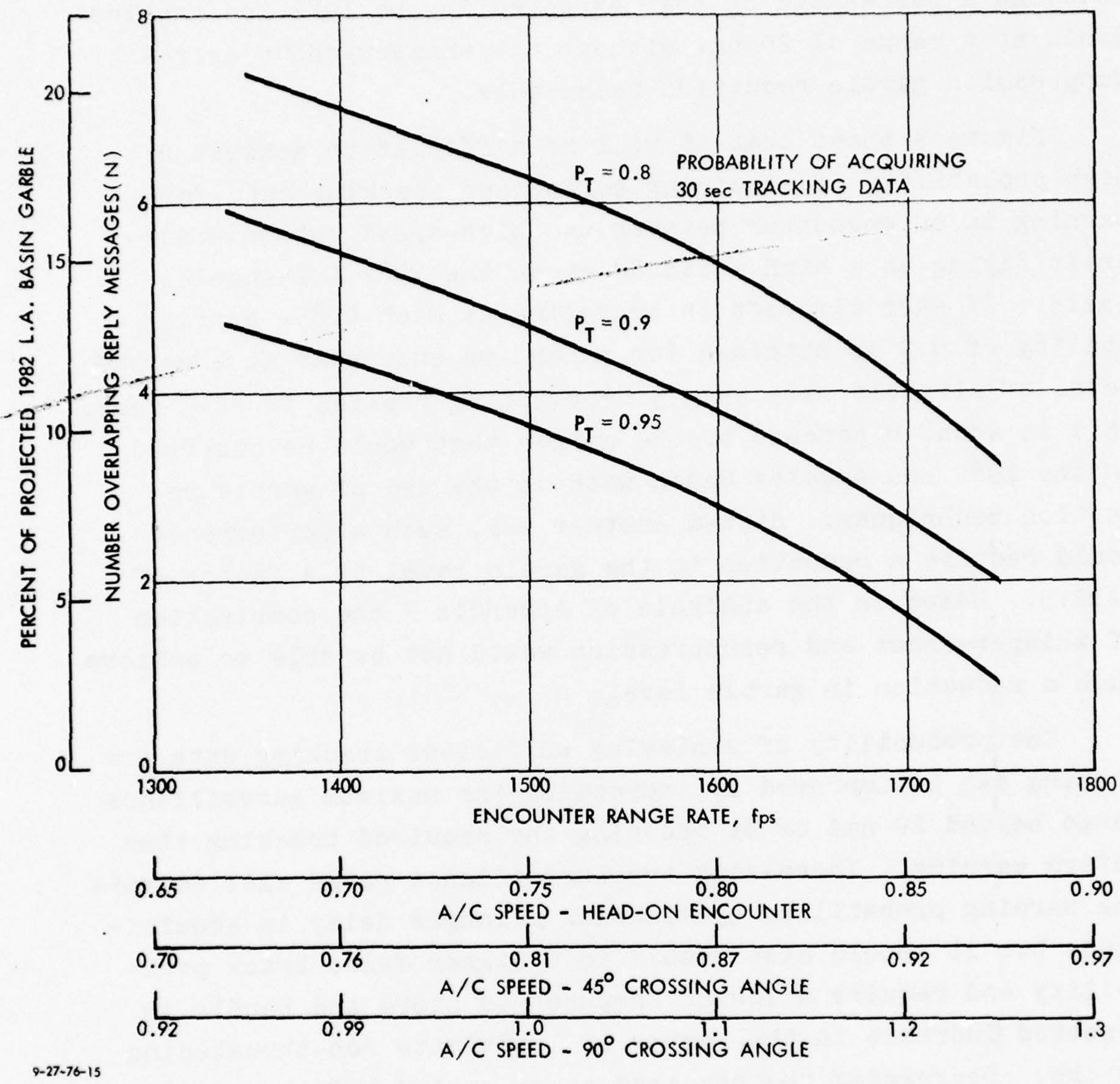


FIGURE 9. MCAS Tracker Performance For Stressing Encounters

the aircraft trajectories. A second scale has also been provided on the ordinate of Fig. 9, which expresses the garble level as a percentage of that expected in the 1982 Los Angeles Basin at a range of 20 nmi without the whisper-shout or resuppression garble reduction techniques.

Figure 9 shows that it will be difficult to achieve a high probability of acquiring sufficient tracking data for warning in an encounter between two high-speed subsonic aircraft flying at a high altitude above the 1982 Los Angeles Basin. If each aircraft is traveling at Mach 0.85, a probability of 0.9 is obtained for a head-on encounter at a garble level of slightly less than 3 overlapping replies in 20.3 μsec. This is about 8 percent of the garble that would be observed in the 1982 Los Angeles Basin without the use of garble reduction techniques. Stated another way, such a performance would require a reduction in the garble level by a factor of 1/12.5. Based on the analysis of Appendix F the combination of whisper-shout and resuppression would not be able to achieve such a reduction in garble level.

The probability of achieving sufficient tracking data for warning can be improved by increasing the maximum surveillance range beyond 20 nmi or by reducing the required tracking time before warning. Increasing the surveillance range will improve the warning probability by allowing a longer delay in acquisition, but it should also result in a higher false track probability and require a larger computer to store and handle an expected increase in the number of legitimate non-threatening tracks. Decreasing the required track period before warning will decrease the likelihood of degarbling the reply altitude codes and should lead to a higher false alarm rate and a reduction in warnings of true threatening tracks.

The probability that an intruder can be acquired in less than the specified number of epochs is equated to the probability

of warning here. This is only approximately correct since the altitude of the intruder must be determined, and it is possible for a track to be acquired but then terminated during the 30-sec tracking period. A track can be terminated if the number of accumulated replies does not satisfy MITRE's track continuation requirements (cf. Section III-E-2) or if a track is pulled off by a crossing aircraft whose range may sweep through the range window of the intruder being tracked (cf. Appendix G).

Only the first of these mechanisms has been considered here (cf. Appendix C - Section D) in estimating the warning delay probability, although the second mechanism has a relatively high associated probability, as shown in Appendix G. Thus, the warning delay estimates given here must be regarded as too low. However, including the possibility of track termination after acquisition but ignoring the possibility of pull-off has a negligible effect on the estimate of the probability of warning.

The probability of track continuation after acquisition is analyzed in Section D of Appendix C. For round reliability of 0.9, 0.8 and 0.7, the probability of track continuation during the 30-sec required tracking period is approximately 1.0, .999 and .988, respectively.

A round reliability of 0.7 is unacceptably low in that it produces too long a delay in acquisition even without considering the possibility of a track termination after acquisition. For a round reliability of 0.8 or higher, the possibility of dropping the track during the required 30-sec tracking period is extremely small, and will not alter the estimate of warning delay.

VI. TRACKING PERFORMANCE

A. GENERAL CONSIDERATIONS

The performance of MCAS depends upon its ability to acquire, update and establish a track on every intruder in its 20-nmi surveillance region and to generate alarms if, and only if, a real intruder penetrates the alarm region defined by the MCAS threat logic. In processing the data acquired during the tracking operation, the system uses a number of criteria and tests (cf. Section III) in order to decide, at each phase, whether an apparent intruder is real and, after a track has been established, whether it is a potential threat. If the tests of the validity of a track are unsuccessful, the MCAS processor may falsely dismiss a real target track or falsely accept a false target track. Even if a real target track is preserved, the estimates of the track parameters may result in an incorrect estimate of the time that the alarm should be given. The latter error is discussed in Section V.

Garble can distort the result of any of these tests and can, therefore, affect tracking performance in several different ways. Acquiring and processing false tracks increases the tracking load, perhaps beyond the system's capacity. Acquired false tracks can persist, in some cases long enough to be established, thereby increasing the probability of false warning beyond that due to measurement errors alone. In addition, in some circumstances, garble can cause the loss of a track, thereby increasing the warning delay beyond what was estimated in Section V, primarily as a result of transponder blockage. By distorting an intruder's communicated altitude data, which are

used not only as part of the threat criteria for warnings but also to help correlate track updates, garble can cause additional false warnings, real track losses, and warning losses.

In Appendix C, a detailed analysis of the effect which garble has on each phase of the tracking operation is given. The results are then used to estimate the probability of false warnings and the expected increase in the MCAS tracking load because of false track acquisitions.

Appendix G analyzes separately the trade-offs which result from available parameter choices in the α-β tracking algorithm (cf. Section III-E-2). The effect on track loss or false warning has not been combined with the estimates obtained in Appendix C because it is not known how MITRE plans to deal with the α-β tracking problems which are caused primarily by the need to accommodate intruders accelerating relative to the MCAS.

B. THE EFFECT OF GARBLE ON MCAS TRACKING OPERATIONS

1. False Track Acquisition

In order to acquire a false track the MCAS must make four false bracket detections. First, there must be two such detections one second apart, with apparent ranges that are consistent with a positive closing rate no greater than 977 kn, so that the range of the ghost intruder appears to have closed by an amount between 0 and 1650 ft.

These ranges are then used to extrapolate the apparent intruder range to an earlier and a later interrogation. The second step in the false track acquisition requires that false brackets be detected at the extrapolated ranges within the allowed tolerance of ± 240 ft.

Finally, the apparent altitude codes, due to garble pulse detections at nominal altitude positions between each of the four false bracket pairs of framing (F_1, F_2) pulses, must pro-

duce a legitimate altitude code when "anded" together. Illegitimate altitude codes are those whose C-bits (cf. Fig. 2) form the binary sequences: 000, 101, and 111. In addition, at least one binary "one" must be obtained among the six A or B bits (A_1, A_2, A_4, B_1, B_2, B_4 for altitudes below 31,000 ft*).

The results of the calculation in Appendix C for false track acquisition indicate that the probability of acquiring a false track increases rapidly with garble. For a number of overlapping replies equal to 3 the expected number of false track acquisitions/sec is 0.056, while for N = 4 it is 1.84, and for N = 5 it is 21.

2. False Track Updating

In the updating process range is extrapolated (cf. Section III-E-2) and the predicted range window (±240 ft centered on the extrapolated range) is then examined for bracket pulse pairs. If a false track has been initiated a subsequent bracket detection in the predicted range window can occur for either of two reasons: the presence of garble or the presence of a legitimate bracket for a reply whose range happens to fall within ±240 ft of the apparent false track range for the given interrogation. In the latter case the same bracket can update the false track as well as the legitimate track.

The track is updated after a bracket detection in the predicted range window if three conditions are satisfied. First, at least one of the three C-bits in the altitude code between the two bracket pulses must be a "one." Second, at least one "one" in the three C-bits of the apparent reply altitude code must agree with at least one "one" of the C-bits in one of the three predicted altitude codes. The three predicted altitudes consist of a best prediction and two adjacent ones

*This assumes that the MCAS aircraft is flying sufficiently lower than 31,000 ft that aircraft with a code corresponding to an altitude above 31,000 ft is not a threat.

(differing by 100 ft). Third, the apparent reply code must pass a correlation test in which a correlation number given by 48-3M-Q must exceed a specified minimum value for one or more of the three predicted altitudes. In this test M is the number of "ones" appearing in the apparent reply code where the predicted altitude code has "zeroes" and Q is the number of "zeroes" appearing in the apparent reply code where the predicted altitude code has "ones."

MITRE has not yet made a final choice of the minimum correlation number required for acceptance of the apparent reply. Settings from 36 to 44 have been tried in computer simulations.

In Appendix C it is shown that a setting below 39 is necessary to avoid rejecting a legitimate reply with only two "ones," i.e., one C-bit and one A or B-bit. On the other hand, with such a setting a reply that consisted entirely of garble pulses and that passed the first two tests would also be accepted when the reply overlap is equal to or greater than four. Thus, it must be concluded that in these cases the correlation number test is redundant; i.e., all garble replies with correlation numbers much smaller than the correlation numbers associated with certain legitimate replies would be eliminated by the first two tests alone.

Assuming that the correlation number test is eliminated, or, equivalently, that the admissible threshold setting is well below 39, the following probabilities that an all-garble altitude code is accepted are obtained in Appendix C. For a number N of overlapping replies equal to 3 the probability is 0.30, for N = 4 the probability is 0.42, and for N = 5 the probability is 0.52.

3. False Track Establishment

MCAS does not generate an alarm until an intruder track is established. This can happen only after 30 interrogations, including the initial 4 which must be accepted for acquisition.

A track will be rejected by MCAS unless a specified minimum accumulated number of replies has been accepted after each of the 26 interrogations between the acquisition and the establishment of a track. The required minimum accumulated numbers, which increase with the number of interrogations, are listed in Table 1 of Section III-E-2.

In Appendix C the probability of establishing a false track, given a false acquisition, is calculated. The probability for the case $N = 5$ is 0.072, for the case $N = 4$ it is 0.0013, and for the case $N = 3$ it is 0.64×10^{-6}.

C. SYSTEM IMPLICATIONS OF FAULTY TRACKING DUE TO GARBLE

1. False Alarms

A false alarm will be generated if the apparent range and range rate associated with an established false track satisfy the single modified tau condition described in Section III-F* and, in addition, the apparent altitude code satisfies altitude threat criteria. The probability that the apparent intruder will satisfy both of these conditions is estimated in Appendix C as 0.0339.

When this probability is combined with the previously calculated false track acquisition rate (Section VI-B-2) and the probability of establishing a false track given a false track acquisition, the result is the false alarm rate. In Appendix C the false alarm rate is calculated in this way with and without the use of the whisper-shout degarbling technique (cf. Section III-G).

*Minimum range alarms are neglected in the false alarm analysis of this report.

Appendix C takes whisper-shout into account by assuming that it divides the total aircraft population into four separate equal groups. For a given number of overlapping replies N, the total number of responding aircraft within 20 nmi when whisper-shout is used will then be four times the number corresponding to that N when whisper-shout is not used. In addition, the false alarm rate corresponding to N when whisper-shout is used will be four times the rate corresponding to the same N when it is not used.

These results are summarized in Table 4. From an inspection of the table it is apparent that when the number of aircraft within a 20 nmi range is as large as 244 the false alarm rate (740/hr) will be unacceptable, even when whisper-shout is used. This number is still considerably short of the 412 aircraft projected for a 20 nmi range about the Los Angeles terminal center in 1982.

TABLE 4. FALSE ALARM RATES AS A FUNCTION OF AIRCRAFT POPULATION*

No. of overlapping replies(N)	Without "whisper-shout": No. of aircraft within 20 nmi	Without "whisper-shout": False alarms per hr	With "whisper-shout": No. of aircraft within 20 nmi	With "whisper-shout": False alarms per hr
3	37	4.4×10^{-6}	148	1.8×10^{-5}
4	49	0.29	196	1.1
5	61	185	244	740

*Extracted from Table C-10 of Appendix C.

2. Tracking Capacity

MCAS maintains a file which stores the updated tracking data for every intruder within the 20-nmi surveillance range. The file also contains data for every acquired false track

until it is dropped as a result of the criteria listed in Table 1 of Section III-E-2.

The capacity of the MCAS track file must be large enough to accommodate the expected number of acquired tracks, real and false, at any given time; otherwise, the system will become saturated and will be obliged to ignore some potential threats. MITRE has proposed that MCAS be designed to handle approximately 200 tracks at a time.

In Appendix C a lower bound on the total track load which the MCAS file must be able to accommodate is calculated. The result for a number N of overlapping replies equal to 3 is 77 tracks, for N = 4 it is 206 tracks, and for N = 5 it is 1726 tracks. Thus, it would appear that the proposed MCAS track file will be saturated when the number of overlapping replies is as large as 4.

3. Impact of Pulse Jitter

In order to achieve reliable altitude pulse detection in the presence of excessive pulse jitter (beyond the ±0.1 μsec of the ATCRBS specification), MITRE has incorporated "or" logic which declares the presence of an altitude pulse when either one of two detections, 0.121 μsec apart, exceeds the detection threshold. When this feature is included in the analysis, in Appendix C, it is found that, for four overlapping replies, the false alarm rate is increased 17 times (relative to the results in Table 4 for four overlapping replies) while the average track load is increased from 206 to 340.

D. RANGE TRACKING

The MITRE BCAS employs the so-called α-β tracker to maintain range tracks after acquisition. The α-β tracker is a computer algorithm which, on the basis of range measurements, estimates range and range rates and predicts these for the subsequent interrogation-reply sequence. Tracker inputs are

range measurements obtained from detected brackets of successive replies, and measurement errors are produced by garble, pulse jitter, and clock quantization errors.

An analysis of the performance of the α-β tracker is given in Appendix G. Errors from multipath and receiver noise are not included in this analysis.

Two mechanisms for range track disruption have been examined in Appendix G: (1) interrogator-intruder relative accelerations encountered during turns such as specified in ANTC-117 (i.e., 0.5g per aircraft), and (2) track pull-off by a crossing aircraft whose range may sweep through the range window of the intruder under track. In the first instance, the combined errors due to acceleration, lag in the tracker and the range measurement errors may place the extrapolated range window (or gate) outside the intruder range. In the second instance, the range track may be transferred, inadvertently, to a crossing aircraft whose reply signal is stronger than that of the intruder and whose range rate is close enough to that of the intruder. Furthermore, if the altitude code of the crossing aircraft is not rejected by the data processing logic then the pull-off remains undetected and the intruder track is lost. Thus, the probability of undetected pull-off during 30 sec (needed for track establishment) prior to alarm is a lower bound* on the probability of false dismissal of legitimate threats.

The basic limitations of BCAS are illustrated by three design cases. In Case A, the tracker is optimized for accelerating encounters; in Case B, the tracker parameters are modified to decrease the probability of undetected pull-off; while Case C examines the consequences of programmed gate size increases. In each case the aircraft population is assumed to

*Undetected pull-off is only one mechanism producing false dismissals.

produce a reply overlap of N = 4. This means that 49 aircraft can be within 20 nmi (instrumented MCAS range) of the interrogator, if no whisper-shout is employed, or 195 aircraft when a four-fold improvement is postulated for whisper-shout. The latter is still below the 412 aircraft projected by FAA for the Los Angeles terminal area in the 1980s. The specific design cases are as follows:

Case A. This case, shown in Table 5, illustrates the performance attainable when the α-β parameters are optimized to minimize the probability of track disruption under accelerating encounters. This probability was thus reduced to a relatively low 4 percent but the probability of undetected pull-off turned out to be a relatively high 19 percent. In a qualitative sense, the result is not unexpected; the range rate difference between a crossing aircraft (with a dominant signal) and the intruder acts as an apparent acceleration of the intruder. Consequently, a tracker designed to follow an accelerating intruder will also tend to follow a dominant crossing aircraft.

Case B, shown in Table 5, illustrates the problems encountered when the α-β parameter pairs are readjusted to decrease undetected pull-offs (results for other pairs are shown in Table G-2 of Appendix G). In this case, the tracker has no capability during accelerations of 0.5g per aircraft, specified by ANTC-117. If, on the other hand, the acceleration does not exceed 0.2g, then the probability of track loss decreases to 50 percent.

TABLE 5. TRACKER PERFORMANCE AS A FUNCTION OF α-β PARAMETERS, USING A ±240 FT WINDOW (Extracted from Table G-4 of Appendix G)

Conditions: (1) Number of overlapping replies = 4

(2) Aircraft population, within 20 nmi, for the above overlap is 49 without "whisper-shout," and 195 with "whisper-shout."

Case A optimizes α-β parameters to minimize track disruption during acceleration.

Case B sacrifices performance during acceleration in order to reduce undetected pull-off probability (other cases are shown in Table 2 of Appendix G).

Tracker performance	Case A $\alpha = 0.4746$ $\beta = 0.6$	Case B $\alpha = 0.1348$ $\beta = 0.05$
Probability of range track disruption, per interrogation, during acceleration*	4%	100%
Probability of undetected pull-off during 30 sec	19%	7.8%

***0.5g turn per aircraft, as specified by ANTC-117.**

Case C postulates gate-size control logic, which upon loss of a reply within the normal gate size, 480 ft, increases the gate to 600 ft. Furthermore, the additional 120 ft are all applied in the direction of the expected tracking error for closing accelerations. Also, the α-β parameters used in Case B were modified to $\beta = 0.1$ and $\alpha = 0.1175$ (corresponding to a tracker settling time of 16 sec), the reasons being that (1) the increased gate size, from 480 to 600 ft produced no significant improvement in Case B performance during accelerating encounters and (2) the change in α-β parameters together with the increased gate size produced some improvement in acceleration tracking with only a slight increase in pull-off probability from 7.8 percent to 8.3 percent. The resultant track loss

probabilities in accelerating encounters were:

1. 8.3 percent when only one aircraft is accelerating at 0.5g.
2. 50 percent or more when both aircraft were accelerating, depending on the relative time and build-up of acceleration.

This represents a significant improvement over Case B. However, the increased gate-size control logic used in Case C impacts the false alarm rate. Any logic, which increases the gate size upon loss of a reply, or replies, is also operative during false tracks and improves the false track survival probability. Whereas, a fixed gate size of 480 ft produced a false alarm rate of 1.1 per hour, the use of increased gates, although limited to 600 ft, produced 62 alarms per hour.

The present study did not uncover any usable α-β parameter pair which can insure reliable tracking in accelerating encounters (0.2g to 0.5g turn per aircraft) without a severe penalty in false dismissals due to undetected track pull-offs during non-accelerating encounters. Attempts to improve tracking through programmed gate size increases, produced very high false alarm rates.

MITRE is experimenting (by computer simulation) with a number of α-β parameter pairs, one of which is represented in Case B in the preceding discussion. Furthermore, some form of programmed gate increase will be included in MCAS to account for range error buildup during track coasts. However, the exact logic which triggers the increase and the size of that increase is not yet clear. In any case, the same logic would be operative during a false track and would therefore increase its survival probability. The ultimate effect is an increase in the false alarm rates above the values computed in Appendix C. The latter analysis is based on a constant range gate, ±240 ft, and consequently provides only a lower bound on the false alarm rate.

VII. CONCLUSIONS

The dominant source of performance limitation for the MCAS is synchronous garble, i.e., mutual interference of ATCRBS transponder replies stimulated by an MCAS interrogations. Asynchronous garble, i.e., interference caused by replies stimulated by other interrogators--both other MCAS and ground secondary surveillance radar (SSR) interrogators--is less serious but still enhances the effects due primarily to synchronous garble. Consequently, garble has dominated the design of MCAS. The design includes direct means to partially suppress the garble and, in addition, includes a sophisticated tracker to overcome the residual garble after partial suppression. The measures of performance of the MCAS in the presence of garble are the rate of generation of false tracks, the probability of dropping real tracks due to tracker saturation or real track interruption, and the probability of delaying the initiation of tracking for a real target.

MITRE has estimated (but has not provided substantiation of the estimate) that the tracker design can operate satisfactorily with a residual synchronous garble level corresponding to that generated by an average of N = 4 aircraft per 1.644 nmi in range. A range increment or range cell of 1.644 nmi corresponds to a round-trip propagation delay of 20.3 μs, which is the length of an ATCRBS transponder reply message. Since the whisper-shout garble suppression technique partitions the aircraft in each range cell into four groups, and the maximum instrumented range of the MCAS is 20 nmi (19.73 nmi = 12 range cells), the maximum number of aircraft that could exist with

N = 4 and perfect whisper-shout partitioning (i.e. four equal groups) would be about 200 aircraft. However, the number of aircraft in a range cell is not expected to be a constant, nor is the partitioning of the aircraft within a range cell expected to be perfect, so that the expected number per partitioned cell may exceed the average number N = 4 in some cases even when the number of aircraft is 200 or less.

For comparison with a total number of 200 aircraft: an FAA static model of the projected traffic in the Los Angeles (LAX) basin for 1982, which was used in previous IDA CAS studies, has 412 aircraft within 20 nmi. The density varies in this model: the average number of aircraft is 10 over the first range cell (0-1.64 nmi), 40 over the range interval from 6.6 to 16.4 nmi (6 range cells), and 34 aircraft in the farthest range cell (18.08-19.73 nmi). It is certain that the MCAS would not perform satisfactorily in this environment by any of the measures cited above unless additional garble suppression improvements are provided.

IDA has found that, at an average value of N = 4 and assuming a garble suppression technique that partitions the population into four groups, the false alarm rate would be about one per hour for 196 aircraft uniformly distributed over 20 nmi. If the traffic increases by 25 percent to 244 the false alarm rate would exceed 740 per hour.

Because of the generation of false tracks for the case N = 4, the expected tracking load for MCAS will be somewhat larger than the 200 tracks that MITRE has estimated as its probable capacity. Thus, it is likely that the system will be often saturated in garble environments any higher than this.

The expected warning delay, in general, should be small enough to provide adequate time for escape maneuvers in a threatening encounter with an intruder that enters the MCAS surveillance region at the 20 nmi range unless the intruder's

closing rate is very high, e.g., higher than about 1500 ft/sec. The warning delay, however, will be too large for MCAS to reach its design goal of responding with adequate time to a 1650 ft/sec closing speed encounter.

Pop-up targets, i.e., intruders that enter the surveillance region at shorter ranges than 20 nmi, will cause warning delay problems. For such intruders, MITRE has proposed relaxing the condition of 30 interrogations (30 sec) including acquisition now required by MCAS for track establishment. MITRE proposes that if four garble-free replies in succession are available that a track be established immediately. This approach does not appear to be feasible in dense traffic because of the high probability of garble replies. The expected false track acquisition rate seems to conflict, in principle, with this kind of solution.

When the intruder is accelerating relative to the MCAS a track may be lost because of range-tracking parameter estimation and range-gate positioning errors. A track may also be lost because of pull-off by a crossing aircraft. If the α-β parameters of the range-tracking algorithm are optimized to insure against track disruption because of acceleration the probability of undetected pull-off will be high. If the parameters are adjusted to reduce the probability of an undetected pull-off the maximum acceleration which MCAS can accommodate will be well below ($\sim$ 0.2g) the 1/2 g per aircraft specified by ANTC-117. An investigation of these effects shows that it is extremely difficult, if not impossible, to select the α-β parameters to provide satisfactory performance simultaneously against both effects.

Range-gate size control logic which would increase the gate, e.g., from 480 ft to 600 ft, when a reply is lost within the normal gate can improve the trade-off between track loss due to acceleration and track loss due to pull-off. However,

it will do so at the cost of a higher false alarm rate because of the resulting increase in the false track survival probability.

The effect of garble in producing false altitude coded replies is not included quantitatively in the MCAS performance estimates given in this report. False altitudes will undoubtedly increase both the false warning rate and warning loss for the system.

REFERENCES

1. Air Transport Association of America, "Airborne Collision Avoidance System," ANTC Report No. 117 (Rev. 10), May 12, 1971. (corrected through September 27, 1971).

2. Westberg, R.N., FAA Memo From EC200 to RD200, "Air Traffic Forecast Data," 27 April 1971.

3. "ATCRBS/Beacon Collision Avoidance System Impact on ATCRBS Performance," ECAC-CR-75-070, September, 1975.

APPENDIX A

U.S. NATIONAL STANDARD FOR AIR TRAFFIC CONTROL RADAR BEACON SYSTEM CHARACTERISTICS

ORDER

DEPARTMENT OF TRANSPORTATION
FEDERAL AVIATION ADMINISTRATION

1010.51A

8 Mar 71

SUBJ: SELECTION ORDER: U. S. NATIONAL AVIATION STANDARD FOR THE MARK X (SIF) AIR TRAFFIC CONTROL RADAR BEACON SYSTEM (ATCRBS) CHARACTERISTICS

1. PURPOSE. This Order revises the ATCRBS Standard in the following respects:

   a. A relaxation in the level of transponder self-test interrogation signal from -70 dBm to -40 dBm.

   b. A relaxation in transponder spurious radiation from -70 dB below one watt to a recommendation that CW spurious radiation be limited to -70 dB below one watt.

   c. Transponder desensitization action has been clarified.

2. CANCELLATION. Order 1010.51 dated 10/10/68 is cancelled.

3. REQUIREMENT. The National Airspace System will utilize ATCRBS, including the revised transponder, as a primary data acquisition source for aircraft position, identity, and pressure-altitude data.

4. SELECTION DECISION. The ATCRBS Standard described in Paragraph 5 of this Order has been shown to be responsive to the requirement stated in Paragraph 3. Accordingly, it is hereby selected for incorporation in the National Airspace System, pursuant to Section 312(c) of the Federal Aviation Act.

5. DESCRIPTION. The attached National Standard for ATCRBS specifies the performance required of each component to meet the overall operational requirements of the common civil/military system. It specifies the technical parameters, tolerances, and techniques to the extent required to ensure proper operation and compatibility between elements of the ATCRBS.

   The Radio Technical Commission for Aeronautics (RTCA) Sub-Committee 116B has completed a report on Minimum Operational Characteristics (MOC) for airborne ATCRBS transponders. This report is not compatible with the National Standard in three areas. An RTCA Ad Hoc Group on Proposed

Distribution: W-1 (minus AS/AT/FS/LG/NS/RD/SM/FI/BU); WAS/AT/FS/LG/NS/RD/SM/FI/BU-3; RAF/AT/AS/FS-2; CFS/PR-2; CAY-3; M-2; [illegible]

Initiated by: RD-242

Changes to the U. S. National Standard on ATCRBS has recommended that the Standard be revised.

The relaxation of transponder self-test interrogation signal and spurious radiation does not conflict with the International Civil Aviation Organization (ICAO) Standards and Recommended Practices for Secondary Surveillance Radar (SSR).

The wording which describes transponder desensitization has been revised in accordance with the RTCA document to clarify the desired action. This does not change the substance of the paragraph from the previous wording.

The attached Standard is revised in accordance with the above recommendations. Paragraphs 2.7.7.1, 2.7.16.2, 2.11.1, and 2.11.1.1 are marked with an asterisk to indicate the changes.

6. INITIAL IMPLEMENTATION CRITERIA. The National Standard for ATCRBS shall be used as the basic document for defining the technical parameters, tolerances, and performance of all ATCRBS components.

7. DIRECTED ACTION. Subject to applicable rulemaking, programming, and budgetary procedures, action shall be taken by the elements of the agency concerned to implement this selection in accordance with the foregoing initial implementation criteria.

J. H. Shaffer
Administrator

U.S. NATIONAL STANDARD FOR THE IFF MARK X (SIF)/AIR TRAFFIC CONTROL RADAR BEACON SYSTEM CHARACTERISTICS

1. GENERAL.

1.1 System Characteristics.

1.1.1 Under Public Law 85-726, the Federal Aviation Administration is charged with providing for the regulation and promotion of civil aviation in such manner as to best foster its development and safety, and to provide for the safe and efficient use of the airspace by both civil and military aircraft, and for other purposes. Explicitly, the Administrator shall develop, modify, test, and evaluate systems, procedures, facilities, and devices, as well as define the performance characteristics thereof, to meet the needs for safe and efficient navigation and traffic control of all civil and military aviation operating in a Common Civil/Military System of Air Navigation and Traffic Control.

1.1.2 A System Characteristic, the logical result of such development effort, specifies the performance required of a component (or subsystem) to meet the overall operational requirements of the System. It specifies the technical parameters, tolerances, and techniques to the extent required to insure proper operation and compatibility between elements of the National Airspace System.

1.1.3 If optimum performance is to be obtained, these System Characteristics must be met by all civil and military users of the Air Traffic Control Radar Beacon System under all expected operating conditions. It is recognized that certain existing equipment does not comply with all requirements of these characteristics. Since such equipment may degrade the quality of service to all users, it is expected that its usage will be phased out as soon as practicable.

1.2 System Characteristics and Guidance Material.

1.2.1 The System Characteristics and Guidance Material provided herein are restricted to those system elements which must be treated in a uniform manner by all concerned, both civil and military, if the IFF Mark X (SIF)/Air Traffic Control Radar Beacon System is to function satisfactorily. In this connection, it is necessary to define closely many characteristics of the airborne component of the system (transponder). The system composed of the Mode 3 portion of the IFF Mark X (SIF) and Modes A and C of the Air Traffic Control Radar Beacon System shall be referred to herein as ATCRBS.

1.2.2. The following are the modes provided by the system, and their associated functions:

Mode 1—For Military use.

Mode 2—For Military use.

Mode 3/A—To initiate transponder response for identification and tracking.

Mode B—In some parts of the world, during a transition period, to initiate transponder response for identification and tracking.

Mode C—To initiate transponder responses for automatic pressure altitude transmission.

Mode D—For future expansion of the system to satisfy operational requirements that may be agreed by the International Civil Aviation Organization. No functional need for Mode D has been defined.

1.2.3 The Air Traffic Control (ATC) System will use Mode 3/A with 4096 identity codes and Mode C with pressure altitude transmission in 100-foot increments in providing separation service to both military and civil aircraft. There

are no plans for use of Modes B and D in the United States.

1.2.4 The ATC System will provide vital support to military operation during periods of national emergency through the continued ATC use of Modes 3/A and C.

1.3 Operational Requirements.

Revised operational requirements for the Common System ATCRBS were originally established by the President's Air Coordinating Committee in Paper ACC 59/20.1-1 dated February 24, 1953, which endorsed the recommendations of the Joint Chiefs of Staff, Joint Communications-Electronics Committee as set forth in their memorandum CECM 58-53, Case 386-G, dated January 15, 1953. These recommendations were subsequently modified by classified correspondence to include a recognition of the 64-code capability of the ATCRBS and to provide for compatibility with the Military IFF Mark X System. Common System Component Characteristics for the ATCRBS were established by the President's Air Coordination Committee in Paper ACC 59/20.4 dated September 1957. Compatible system characteristics were approved by the International Civil Aviation Organization (ICAO), Sixth Communication Division, and published in the International Standards and Recommended Practices Aeronautical Telecommunications, Annex 10, Fifth Edition dated October 1958. Three-pulse side lobe suppression, automatic pressure-altitude transmission and other improvements were recommended by the ICAO Seventh Communications Division and included in the report of the Seventh Session dated February 9, 1962. At the ICAO COM/OPS Meeting in 1966, the three-pulse method was designated as the sole means of side lobe suppression and 4096 identity codes were raised to Standards. A standard of correspondence (paragraph 2.7.13.2.5) was established for automatic pressure-altitude transmission and a functional description of the modes and their intended usage was defined.

1.3.1 Compatibility. The required compatibility of the Military Mark X (SIF) airborne transponders with the ICAO SSR (ATCRBS) has been established using the Military Mode 3 and Civil Mode A which are identical in characteristics. This mode of operation is referred to herein as Mode 3/A. The Memorandum of Understanding between the Department of Defense and the Federal Aviation Administration on the Joint Operational Use of the Military IFF Mark X (SIF) System and the Common System Air Traffic Control Radar Beacon System, dated March 18, 1966, contains the agreement on the use of Modes 3/A and C.

1.3.2 System Coverage. The ATCRBS is intended to provide the air traffic controller with continuous, reliable, and accurate information concerning the plan-position (rho-theta), altitude, and identity of any or all equipped aircraft in the airspace under his control. With a properly sited Air Traffic Control Radar Beacon Interrogator-Receiver and other units having characteristics as stated herein, the ATCRBS will provide spatial line-of-sight coverage equal to or greater than the following limits:

a. Range .... 1 to 200* nautical miles

b. Elevation .... ½ to 45° above the horizontal plane

c. Altitude .... Limited only by service ceiling of aircraft

*Interrogators having limited range may be employed at many locations.

While it is necessary to establish specific standards for the airborne components of the System and to define the characteristics of the radiated signals from both the interrogator and transponder, the power and sensitivity requirements for the interrogator-receiver may be modified on the basis of the intended usage with due regard for the precautions outlined in the guidance material.

1.3.3 System Accuracy. The system accuracy is determined by the characteristics of the ATC beacon interrogator-receiver (including its antenna), transponder, altimeter and transducer, ground processing equipment, and the associated display. With equipment of present day design meeting the characteristics stated herein, ATCRBS is capable of providing data within the following accuracies:

a. Range: ±1000 feet.

b. Azimuth: ±1.0 degree

c. Altitude Correspondence: Within ±125 feet, on a 95 percent probability basis, with

the pressure altitude information (referenced to the standard pressure setting of 29.92 inches of mercury) used on board the aircraft to adhere to the assigned flight profile.

1.3.4 Identification Coding

1.3.4.1 The ATCRBS is a valuable tool for identifying aircraft, as well as for providing radar target reinforcement.

The inherent capability of the system to provide radar identification of participating aircraft will be utilized to provide the controller with the specific radar beacon target identity of those aircraft equipped. The characteristics specified herein provide for 4096 discrete reply codes. In addition to the 4096 discrete reply codes, a Special Position Identification (SPI) pulse is available for transmission upon request of the control agency, on any mode except Mode C.

1.3.4.2 Two codes shall be reserved for transmission of distinct emergency and radio communications failure indications.

1.3.4.2.1 Code 7700 shall be used on Mode 3/A to provide recognition of an aircraft in an emergency.

NOTE.—Some existing military transponders transmit four trains of the code in use as an emergency reply. Other military transponders transmit the code in use followed by three trains of Code 0000 as the emergency reply. New military transponders will transmit Code 7700 followed by three trains of Code 0000 as an emergency reply.

1.3.4.2.2 Code 7600 shall be used on Mode 3/A to provide recognition of an aircraft with radio communications failure.

1.3.5 Altitude Transmission. The system provides for automatic pressure-altitude data transmission in 100-foot increments from −1000 feet to 126,700 feet.

1.3.5.1 This pressure altitude data transmission capability will be used to:

a. Reduce the volume of communications between controllers and pilots by obviating the need for oral altitude reports.

b. Improve utilization of airspace in connection with the provision of ATC services to climbing and descending aircraft.

c. Enable the controller, when desirable, to assure himself that vertical separation between two aircraft is being maintained.

d. Provide ATC an improved means of determining when greater vertical separation is required due to turbulence.

e. Improve the integrity of the Air Traffic Control Radar Beacon System (ATCRBS) for ATC purposes by automatically displaying to the controller the targets and altitudes or flight levels of aircraft in or near the airspace under his jurisdiction which are not otherwise selected for display.

f. Reduce the number of advisories and traffic avoidance vectors required in the provision of radar traffic information and vectoring service.

g. Improve ATC efficiency in serving high performance aircraft during cruise-climb profiles.

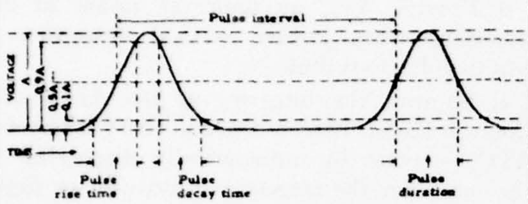
h. At a future date, enable automatic prediction of projected flight conflicts in elevation using electronic data processing systems.

1.4 ATCRBS System Description.

The System consists of airborne transponders, ground interrogator-receiver, processing equipment, and an antenna system. The antenna may or may not be associated with, or slaved to, a primary surveillance radar. In operation, an interrogation pulse-group transmitted from the interrogator-transmitter unit, via an antenna assembly, triggers each airborne transponder located in the direction of the main beam, causing a multiple pulse reply group to be transmitted from each transponder. These replies are received by the ground receiver and, after processing, are displayed to the controller. Measurement of the round-trip transit time determines the range (rho) to the replying aircraft while the mean direction of the main beam of the interrogator antenna, during the reply, determines the azimuth (theta). The arrangement of the multiple-pulse reply provides individualized pressure altitude and identity information pertaining to the responding aircraft. The ATCRBS is the preferred means of obtaining aircraft three dimensional position and identification data in the National Airspace System.

2. SYSTEM CHARACTERISTICS.

Pulse Shape and Spacing Definitions



DEFINITIONS

Pulse amplitude A. The peak voltage amplitude of the pulse envelope.

Pulse duration. The time interval between 0.5A points on leading and trailing edges of the pulse envelope.

Pulse interval. The time interval between the 0.5A point on the leading edge of the first pulse and the 0.5A point on the leading edge of the second pulse.

Pulse rise time. The rise time as measured between 0.1A and 0.9A on the leading edge of the pulse envelope.

Pulse decay time. The decay time as measured between 0.9A and 0.1A on the trailing edge of the pulse envelope.

2.1 Interrogation and Control (Interrogation Side Lobe Suppression) Radio Frequencies (ground-to-air).

2.1.1. Center frequency of the interrogation and control transmissions shall be 1030 MHz.

2.1.1.1 The frequency tolerance shall be ±0.2 MHz.

2.1.2 Center frequencies of the control transmission and each of the interrogation pulse transmissions shall not differ from each other by more than 0.2 MHz.

2.2 Reply Radio Frequency (air-to-ground).

2.2.1 Center frequency of the reply transmission shall be 1090 MHz.

2.2.1.1 The frequency tolerance shall be ±3 MHz.

2.3 Polarization.

2.3.1 Polarization of the interrogation, control, and reply transmissions shall be predominantly vertical.

2.4 Interrogation Modes (signals-in-space).

2.4.1 The interrogation shall consist of two transmitted pulses designated P_1 and P_3. A control pulse P_2 shall be transmitted following the first interrogation pulse P_1.

2.4.2 The interrogation modes shall be as defined in 2.4.3.

2.4.3 The interval, between P_1 and P_3, shall determine the mode of interrogation and shall be as follows:

Mode 1	3±0.1 Microseconds
Mode 2	5±0.2 Microseconds
Mode 3/A	8±0.2 Microseconds
Mode B	17±0.2 Microseconds
Mode C	21±0.2 Microseconds
Mode D	25±0.2 Microseconds

2.4.4 The interval between P_1 and P_2 shall be 2.0±0.15 microseconds.

2.4.5 The duration of pulses P_1, P_2, and P_3 shall be 0.8±0.1 microsecond.

2.4.6 The rise time of pulses P_1, P_2, and P_3 shall be between 0.05 and 0.1 microsecond.

NOTE.—The intent of the lower limit of rise time (0.05 microsecond) is to reduce sideband radiation. Equipment will meet this requirement if the sideband radiation is no greater than that which theoretically would be produced by a trapezoidal wave having the stated rise time.

2.4.7 The decay time of pulses P_1, P_2, and P_3 shall be between 0.05 and 0.2 microsecond.

NOTE.—The intent of the lower limit of decay time (0.05 microsecond) is to reduce sideband radiation. Equipment will meet this requirement if the sideband radiation is no greater than that which theoretically would be produced by a trapezoidal wave having the stated decay time.

2.5 Interrogation and Side Lobe Suppression Transmission Characteristics (signals-in-space).

2.5.1 The system relies on pulse amplitude comparison between pulses P_1 and P_2 in the transponder to prevent response to side lobe interrogation. The radiated amplitude of P_2 at the antenna of the transponder shall be (1) equal to or greater than the radiated amplitude of P_1 from the greatest side lobe transmission of the antenna radiating P_1, and (2) at a level lower than 9 dB below the radiated amplitude of P_1 within the desired arc of interrogation, (see 3.2.2).

2.5.2 Within the desired arc of the directional interrogation (main lobe), the radiated amplitude of P_3 shall be within 1 dB of the radiated amplitude of P_1.

2.6 Reply Transmission Characteristics (signals-in-space).

2.6.1 Framing Pulses. The reply function shall employ a signal comprising two framing pulses spaced 20.3 microseconds, as the most elementary code.

2.6.2 Information Pulses. Information pulses shall be spaced in increments of 1.45 microseconds from the first framing pulse. The designation and position of these information pulses shall be as follows:

Pulse	*Position Microseconds*
C_1	1.45
A_1	2.90
C_2	4.35
A_2	5.80
C_4	7.25
A_4	8.70
X	10.15
B_1	11.60
D_1	13.05
B_2	14.50
D_2	15.95
B_4	17.40
D_4	18.85

NOTE.—The Standard relating to the use of these pulses is given in 1.3.4 and 2.7.13. However, the position of the "X" pulse is specified only as a technical standard to safeguard possible future use. *Further guidance on this matter is given in 3.3.6.*

2.6.3 Special Position Identification (SPI) Pulse. In addition to the information pulses provided, a Special Position Identification pulse, which may be transmitted with the information pulses, shall occur at a pulse interval of 4.35 microseconds following the last framing pulse.

2.6.4 Reply Pulse Shape. All reply pulses shall have a pulse duration of 0.45±0.10 microsecond, a pulse rise time between 0.05 and 0.1 microsecond, and a pulse decay time between 0.05 and 0.2 microsecond. The pulse amplitude variation of one pulse with respect to any other pulse in a reply train shall not exceed 1 dB.

NOTE.—*The intent of the lower limit of rise and decay times (0.05 microsecond) is to reduce side-band radiation.* Equipment will meet this requirement if the sideband radiation is no greater than that which theoretically would be produced by a trapezoidal wave having the stated rise and decay times.

2.6.5 Reply Pulse Interval Tolerances. The *pulse interval tolerance for each pulse (including* the last framing pulse) with respect to the first framing pulse of the reply group shall be ±0.10 *microsecond.* The *pulse interval tolerance of the* Special Position Identification Pulse with respect to the last framing pulse of the reply group shall be ±0.10 *microsecond.* The pulse interval tolerance of any pulse in the reply group with respect to any other pulse (except the first framing pulse) shall not exceed ±0.15 microsecond.

2.6.6 Code Nomenclature. The code designations shall consist of four digits, each of which lies between 0 and 7, inclusive, and is determined by the sum of the pulse subscripts given in 2.6.2, employed as follows:

Digit	*Pulse Group*
First (most significant)	A
Second	B
Third	C
Fourth	D

2.6.6.1 Examples:

a. Code 3600 would consist of information pulses A_1, A_2, B_2, and B_4.

b. Code 2057 would consist of A_2, C_1, C_4, D_1, D_2, and D_4.

c. *Code 0301 would consist of* B_1, B_2, *and* D_1.

2.7 Technical Characteristics of the Airborne Transponder.

2.7.1 Reply. When selected to reply to a particular interrogation mode, the transponder shall reply (not less than 90% efficiency) when all of the following conditions have been met:

2.7.1.1. The received amplitude of P_3 is in excess of a level 1 dB below the received amplitude of P_1 but no greater than 3 dB above the received amplitude of P_1.

2.7.1.2 Either the received amplitude of P_1 is in excess of a level 9 dB above the received amplitude of P_2, or no pulse is received at the position 2±0.7 microseconds following P_1.

2.7.1.3 The received amplitude of a proper interrogation is more than 10 dB above the received amplitude of random pulses where the latter are not recognized by the transponder as P_1, P_2, or P_3.

2.7.2 No Reply. The Transponder shall not

reply to more than 10% of the interrogations under the following conditions:

2.7.2.1 To interrogations when the interval between pulses P_1 and P_3 differs from that specified in 2.4.3 for the mode selected in the transponder by more than ± 1.0 microsecond.

2.7.2.2 Upon receipt of any single pulse which has no amplitude variations approximating a normal interrogation condition.

2.7.3. Dead Time. After reception of a proper interrogation, the transponder shall not reply to any other interrogation at least for the duration of the reply pulse train. This dead time shall end no later than 125 microseconds after the transmission of the last reply pulse of the group.

2.7.4 Suppression. Upon receipt of an interrogation, complying with 2.4 in respect of the mode selected manually or automatically, the transponder shall be suppressed (not less than 99% efficiency) when the received amplitude of P_2 is equal to or in excess of the received amplitude of P_1, and spaced 2 ± 0.15 microseconds.

NOTE.—It is not the intent of this paragraph to require the detection of P_3 as a prerequisite for initiation of suppression action.

2.7.4.1 The transponder suppression shall be for a period of 35 ± 10 microseconds.

2.7.4.2 The suppression shall be capable of being reinitiated for the full duration within two microseconds after the end of any suppression period.

2.7.5 Receiver Sensitivity and Dynamic Range.

2.7.5.1 The minimum triggering level (MTL) of the transponder shall be such that replies are generated to 90% of the interrogation signals when:

2.7.5.1.1 The two pulses P_1 and P_3 constituting an interrogation are of equal amplitude and P_2 is not detected; and,

2.7.5.1.2 The amplitude of these signals received at the antenna end of the transmission line of the transponder is nominally 71 dB below 1 milliwatt with limits between 69 and 77 dB below 1 milliwatt.

NOTE.—For this MTL requirement, a nominal 3 dB transmission line loss and an antenna performance equivalent to that of a simple quarter wave antenna are assumed. In the event these assumed conditions do not apply, the MTL of the installed transponder system must be comparable to that of the assumed system.

2.7.5.2 The variation of the minimum triggering level between modes shall not exceed 1 dB for nominal pulse spacings and pulse widths.

2.7.5.3 The reply characteristics shall apply over a received signal amplitude range between minimum triggering level and 50 dB above minimum triggering level.

2.7.5.4 The suppression characteristics shall apply over a received signal amplitude range between 3 dB above minimum triggering level and 50 dB above minimum triggering level.

2.7.6 Pulse Duration Discrimination. Signals of received amplitude between minimum triggering level and 6 dB above this level, and of a duration less than 0.3 microsecond at the antenna, shall not cause the transponder to initiate more than 10% reply or suppression action. With the exception of single pulses with amplitude variations approximating an interrogation, any single pulse of a duration more than 1.5 microseconds shall not cause the transponder to initiate reply or suppression action over the signal amplitude range of minimum triggering level (MTL) to 50 dB above that level.

2.7.7 Echo Suppression and Recovery. The transponder shall contain an echo suppression facility designed to permit normal operation in the presence of echoes of signals in space. The provision of this facility shall be compatible with the requirements for suppression of side lobes given in 2.7.4.

2.7.7.1 Desensitization. Upon receipt of any pulse more than 0.7 microsecond in duration, the receiver shall be desensitized by an amount that is within at least 9 dB of the amplitude of the desensitizing pulse, but shall at no time exceed the amplitude of the desensitizing pulse, with the exception of possible overshoot during the first microsecond following the desensitizing pulse. Single pulses of duration less than 0.7 microsecond are not required to cause the specified desensitization, and shall not cause desensitization of duration greater than that permitted herein or by 2.7.7.2.

2.7.7.2 Recovery. Following desensitization, the receiver shall recover sensitivity (within 3 dB of minimum triggering level) within 15 microseconds after reception of a desensitizing pulse having a signal strength up to 50 dB above minimum triggering level. Recovery shall be nominally linear at an average rate not exceeding 3.5 dB per microsecond.

NOTE.—Transponders that respond to military modes will recover within 15 microseconds, but may employ methods other than nominally linear recovery.

2.7.8 Random Triggering Rate. Installation in the aircraft shall be made in such manner that, with all possible interfering equipments installed in the same aircraft operating in their normal manner on operational channels of maximum interference, but with the absence of bona fide interrogations, the random triggering rate (squitter) of the transponder shall not exceed thirty replies per second as integrated over an interval equivalent to at least 300 random triggers, or 30 seconds, whichever is less.

2.7.9. Interference Suppression Pulses. If the equipment is designed to accept and respond to suppression pulses from other electronic equipment in the aircraft (to disable it while the other equipment is transmitting), the equipment must regain normal sensitivity, within 3 dB, not later than 15 microseconds after the end of the applied suppression pulse.

2.7.10 Reply Rate.

2.7.10.1 For equipment intended for installation in aircraft which operate at altitudes above 15,000 feet, the transponder shall be capable of at least 1200 replies per second for a 15-pulse coded reply.

2.7.10.2 For equipment intended for installation in aircraft which operate at altitudes not exceeding 15,000 feet, the transponder shall be capable of at least 1,000 replies per second for a 15-pulse coded reply.

2.7.10.3 A sensitivity reduction type reply rate limit control shall be incorporated in the transponder. The range of this control shall permit adjustment, as a minimum, to any value between 500 and 2000 replies per second, or to the maximum reply rate capability if less than 2000 replies per second, without regard to the number of pulses in each reply. Sensitivity reduction in excess of 3 dB shall not take effect until 90% of the selected value is exceeded. Sensitivity reduction shall be at least 30 dB for rates in excess of 150% of the selected value.

2.7.10.3.1 Recommendation. The reply rate limit should be set at 1200 replies per second, or the maximum value below 1200 replies per second of which the transponder is capable (see 2.7.10.2).

2.7.11 Reply Delay and Jitter. The time delay between the arrival at the transponder of the leading edge of P_3, and the transmission of the leading edge of the first pulse of the reply shall be 3±0.5 microseconds. The total jitter of the reply pulse code group, with respect to P_3, shall not exceed ±0.1 microsecond for receiver input levels between 3 and 50 dB above minimum triggering level. Delay variations between modes on which the transponder is capable of replying shall not exceed 0.2 microsecond.

2.7.12 Transponder Power Output.

2.7.12.1 For equipment intended for installation in aircraft which operate at altitudes above 15,000 feet, the peak pulse power available at the antenna end of the transmission line of the transponder shall be at least 21 dB and not more than 27 dB above 1 watt.

2.7.12.2 For equipment intended for installation in aircraft which operate at altitudes not exceeding 15,000 feet, the peak pulse power available at the antenna end of the transmission line of the transponder shall be at least 18.5 dB and not more than 27 dB above 1 watt.

NOTE.—For the power output requirement of 2.7.12.1 and 2.7.12.2, a nominal 3 dB transmission line loss and an antenna performance equivalent to that of a simple quarter-wave antenna are assumed. In the event these assumed conditions do not apply, the peak pulse power of the installed transponder system must be comparable to that of the assumed system.

2.7.13 Reply Codes.

2.7.13.1 Identification. The 4096 codes available in the Standard at 2.6.2 shall be manually selectable for reply to interrogations on Mode 3/A.

2.7.13.2 Pressure-Altitude Transmissions.

2.7.13.2.1 Independently of the other modes

and codes manually selected, the transponder shall automatically reply to Mode C interrogations.

NOTE.—Military transponders may include provisions to disable Mode C replies.

2.7.13.2.2 The reply to Mode C interrogations shall consist of the two framing pulses specified in 2.6.1 together with information pulses specified in 2.6.2.

2.7.13.2.3 At as early a date as practicable, transponders shall be provided with means to remove the information pulses, but to retain the framing pulses when the provision of 2.7.13.2.6 is not complied with, in reply to Mode C interrogation.

NOTE.—The information pulses should be capable of being removed either in response to a failure detection system or manually at the request of the controlling agency.

2.7.13.2.4 The information pulses shall be automatically selected by an analog-to-digital converter connected to a pressure-altitude data source in the aircraft referenced to the standard pressure setting of 29.92 inches of mercury.

2.7.13.2.5 Pressure altitude shall be reported *in 100-foot increments by selection of pulses* as shown in Figure 1.

NOTE.—Some transponders in service transmit the Special Position Identification (SPI) pulse in addition to the D_4 pulse.

2.7.13.2.6 The digitizer code selected shall correspond to within ±125 feet, on a 95 percent probability basis, with the pressure altitude information (referenced to the standard pressure setting of 29.92 inches of mercury) used on board the aircraft to adhere to the assigned flight profile.

NOTE.—Guidance material relating to pressure altitude transmission is contained in 3.3.4 and 3.3.5.

2.7.14 Transmission Time of Special Position Identification (SPI) Pulse. When manually selected, the SPI pulse shall be transmitted for a period of between 15 and 30 seconds and must be capable of being reinitiated at any time.

2.7.15 Transponder Receiver Bandwidth. The skirt bandwidth should be such that the sensitivity of the transponder is at least 60 dB down at frequencies outside the band 1030±25 MHz.

2.7.16 Transponder Self-Test and Monitor.

2.7.16.1 Self-test and monitor devices that radiate test interrogation signals, or prevent transponder reply to proper interrogation during the test period, shall be limited to intermittent use which is no longer than required to determine the transponder status.

2.7.16.2 The test interrogation rate shall not exceed 450 per second and the test interrogation signal level at the antenna end of the transmission line shall not exceed a level of −40 dBm.

2.7.17 Antenna.

2.7.17.1 The transponder antenna system, when installed on an aircraft, shall have a radiation pattern which is essentially omni-directional in the horizontal plane.

2.7.17.2 Recommendation. The vertical beamwidth (half power points) should be at least 30 degrees above and below the horizontal plane.

NOTE.—Guidance material relating to airborne transponder antennas is contained in 3.3.2.

2.8 Technical Characteristics of the Interrogator-Receiver.

2.8.1 Interrogation Repetition Frequency. The maximum interrogation repetition frequency shall be 450 interrogations per second.

NOTE.—This value is the sum total of the interrogation rate of all modes in use.

2.8.1.1 Recommendation. To minimize unnecessary transponder triggering and the resulting high density of mutual interference, all interrogators should use the lowest practicable interrogation repetition frequency that is consistent with the display characteristics, interrogator antenna bandwidth, and antenna rotation speed employed.

2.8.2 Power Output.

2.8.2.1 The effective radiated peak power of interrogation pulses (P_1 and P_3) shall not exceed 52.5 dB above one watt, and P_1 shall be within 1 dB of P_3.

NOTE.—The effective radiated peak power includes the antenna gain and the transmission line losses. The effective radiated peak power of interrogation should be the minimum required to provide the system coverage. The system coverage stated in 1.3.2 can be met by an interrogator having a nominal 1000 watts power (peak pulse), a transmission line loss of 3 dB, and an antenna gain of 21 dB.

2.8.2.2 Interrogators with range coverage requirements of less than 200 miles will be employed at many locations. The effective radiated peak power of interrogation at these sites shall be reduced to the minimum required level which is practical to achieve.

2.8.3 Receiver Sensitivity.

2.8.3.1 The maximum receiver sensitivity shall be not less than 85 dB below one milliwatt, to produce a tangential signal output, for a 200-mile facility.

NOTE.—For this receiver sensitivity requirement, a nominal 3 dB transmission line loss and an antenna gain of 21 dB are assumed. In the event these assumed conditions do not apply, the receiver sensitivity of the installed system should be comparable to that of the assumed system.

2.8.3.2 Interrogators with range coverage requirements of less than 200 miles will be employed at many locations. The maximum receiver sensitivity at these sites may be reduced to the minimum required level.

2.8.4 Sensitivity Time Control (STC). The receiver sensitivity shall be reduced at short ranges to minimize reply path reflections and pulse stretching. At 15.36 microseconds after the leading edge of pulse P_3 (1 nautical mile plus transponder delay), the gain shall be reduced to an adjustable value between 10 and 50 dB below maximum sensitivity. The recovery rate shall be adjusted to suit local conditions.

2.8.4.1 Recommendation. Following the initial reduction of sensitivity at 15.36 us after the leading edge of pulse P_3 (1 nautical mile plus transponder delay), a recovery rate of 6 dB for each doubling of range is satisfactory for most applications.

2.8.5 Receiver Bandwidth and Video Response. The bandwidth of the receiver shall be centered on 1090 MHz and shall be adequate to reproduce the transponder pulse train described in paragraph 2.6 and to accommodate the transponder transmitter frequency tolerance and interrogator receiver local oscillator drift. The bandwidth shall not be more than 24 MHz at 40 dB below maximum sensitivity. The video response shall be capable of reproducing the pulse trains described in paragraph 2.6 without appreciable pulse stretching or distortion over a dynamic range from receiver threshold to a level 24 dB above threshold, at any range with STC provisions operative.

2.8.5.1 Image Response. The image response shall be at least 60 dB below maximum sensitivity.

2.8.6 Azimuth Accuracy. The electrical alignment of the main lobe of the directional antenna radiation pattern, with respect to the associated display shall be such as to permit received replies to be displayed within 1 degree of true orientation.

2.9 Interrogator Radiated Field Pattern. Recommendation—The beamwidth of the directional interrogator antenna should not be wider than is operationally required. The side- and back-lobe radiation of the directional antenna should be at least 24 dB below the peak of the main-lobe radiation.

2.10 Interrogator Monitor.

2.10.1 The range and azimuth accuracy of the ground interrogator shall be monitored continuously.

NOTE.—Interrogators that are associated with and operated in conjunction with primary radar may use the primary radar as the monitoring device; alternatively an electronic range and azimuth accuracy monitor would be required.

2.10.2 Recommendation. In addition to range and azimuth monitoring, provision should be made to monitor continuously the other critical parameters of the ground interrogator for any degradation of performance exceeding the allowable system tolerances and to provide an indication of any such occurrence.

NOTE.—Guidance on those system parameters for which continuous or periodic monitoring provisions are of particular importance is to be found in paragraph 3.2.5.

2.11 Spurious Emissions and Spurious Responses.

2.11.1 Spurious Radiation. Spurious radiation shall not exceed 76 dB below 1 watt for the interrogator.

2.11.1.1 Recommendation. CW radiation should not exceed 70 dB below 1 watt for the transponder.

2.11.2 Spurious Responses. The response of both airborne and ground equipment to signals not within the receiver bandpass shall be at least 60 dB below maximum sensitivity.

3. GUIDANCE MATERIAL RELATED TO THE AIR TRAFFIC CONTROL RADAR BEACON SYSTEM CHARACTERISTICS

3.1 Factors Affecting Optimum Utilization of the System. A number of specific precautions may be taken in order to obtain maximum utilization of the ATCRBS system, such as:

3.1.1 Coordination of the number and type of interrogators installed in any particular area, and cooperative use of interrogators, where possible, for several related functions.

3.1.2 Use for each interrogator of the lowest interrogation rate which is required to perform its function.

3.1.3 Use for each interrogator of the lowest power output which is required for it to provide satisfactory performance.

3.1.4 Use of particular interrogators only during the periods necessary for them to perform their function.

3.1.5 Limitation of antenna beamwidth to the minimum required and use of low-side-lobe antennas.

3.1.6 Coordination of the interrogation repetition frequency used to minimize interference.

3.2 Application Considerations of the ATCRBS System.

3.2.1 Siting. Care should be taken in siting the ground interrogator to ensure that the number of ground installations is kept to a minimum consistent with the operational requirement for ATCRBS information. It should be emphasized that the effects obtained by reflection of the main lobe are more serious than those associated with primary radar. It is therefore necessary to ensure that no large vertical reflecting surfaces are within a reasonable distance of the ATCRBS interrogator antenna. This distance will depend on the area of the reflecting surface and its elevation with respect to the interrogator, but as a guide, it is desirable to site the interrogator at least half a mile away from large metal structures. Although it may be desirable to associate the interrogator antenna physically with a primary radar antenna, siting and maintenance considerations may make it necessary to have a separate site for the interrogator. When this is necessary, the rotation of the two antennas should be synchronized with a maximum error not to exceed one degree.

3.2.2 Interrogator Antenna.

3.2.2.1 It is necessary that the side lobe level of the antenna relative to the main lobe be as low as practicable. A level lower than −24 dB is desirable. It is important that the interrogation beamwidth be kept as narrow as possible, normally of the order of 3 degrees, but it should be noted that there is a minimum number of replies necessary for processing and display. This minimum will depend on the particular processing and display facilities provided, but would, typically, fall in the range of 4 to 8 replies per beamwidth on each interrogation mode.

3.2.2.2 Side lobe suppression requires two radiation patterns: A directional pattern to radiate the interrogation pulses, and an omnidirectional pattern to radiate the control pulse. It should be noted that "omni-directional", as used here, assumes that adequate power is radiated in all directions, not necessarily that equal power is radiated in all directions. It is necessary that the control pattern be in the right relationship to the interrogation pattern over the operational angles of elevation. This may demand that the antennas be designed in a common assembly so that the same effective height above the ground can be maintained for both.

3.2.2.3 Some antenna sites may experience severe reflections from buildings and re-siting may not be practicable. If the reflections are not adequately suppressed by side lobe suppression, satisfactory performance is possible by use of modified three-pulse side lobe suppression techniques. One technique makes use of the omni-directional antenna during transmission of the P_1 interrogation pulse. The P_1 interrogation is fed to both the directional and omni-directional antennas in a power ratio depending on the particular reflection problem and assists transponder suppression in the side lobe areas.

3.2.3 Sensitivity Time Control (STC). This feature is extremely effective in minimizing the undesirable processing and display of side lobe replies from older transponders that do not have side lobe suppression (SLS) capability. Even with SLS fully implemented, the use of STC will be required to minimize the effects of re-

flected signals and pulse stretching. The setting of STC is critical since too much attenuation will cause target loss and too little allows reflection and side lobe breakthrough. Once an optimum setting is determined, it should be maintained with close tolerance. A tolerance of ±1.5 dB is recommended.

3.2.4 Rejection of Unwanted Responses.

3.2.4.1 In an area where a large number of ground interrogators are necessary, there will be a considerable number of transponder responses, which have been triggered by other interrogators, received at any one ground equipment. The responses will be received at recurrence frequencies which will, in all probability, be different from that of the interrogator receiving the information and will constitute a nuisance called "fruit" (unsynchronized replies) on the radar display.

3.2.4.2 Defruiting techniques which use delay lines, storage tubes, or digital storage to defruit on a pulse-to-pulse basis should be employed to remove these non-synchronous replies. The defruiting function may also be an integral part of the digital detection process.

3.2.5 Monitoring of ATCRBS Interrogator.

3.2.5.1 The performance monitoring of the ground interrogator called for in 2.10 is required to provide responsible personnel with an indication that the equipment is functioning satisfactorily within the system limits and to give an immediate indication of any significant fault developing in the equipment. Additionally, it is desirable that continuous monitoring provisions with respect to at least the system parameters listed hereafter in 3.2.5.1.1 and 3.2.5.1.2 be provided and that an alarm indication be given in the event of a failure of the monitor itself.

3.2.5.1.1 Pulse Intervals. Means should be provided to measure pulse spacings for all modes which are to be employed.

3.2.5.1.2 Interrogator Relative Radiated Pulse Levels. When side lobe suppression is provided, monitoring of this parameter is most important and should be associated with the tolerances indicated in 2.5.

3.2.5.2 Monitoring of the following ATCRBS system parameters is also desirable; however, checking on a periodic basis should suffice.

3.2.5.2.1 Interrogator Radio Frequencies. Assuming that a high stability crystal controlled oscillator is used as the frequency control element of the ATCRBS, it will be necessary only on a periodic basis to determine that the tolerances specified in 2.1 are satisfied.

3.2.5.2.2 Interrogator Pulse Duration.

3.2.5.2.3 Receiver Sensitivity.

3.2.5.2.4 *Radiated Power.*

3.2.5.2.5 Spurious Radiation.

3.2.5.3 The precise location of the monitor warning indication is a matter for determination by the Administration concerned in the light of local circumstances, but should take into account the need to prevent the presentation of erroneous information to the controller without his knowledge.

3.3 Airborne Equipment.

3.3.1 Transponder Peak Power Output and Sensitivity. System requirements can be met by a transponder having a nominal output power of 500 watts (peak pulse) and a nominal minimum triggering level (MTL) of −74 dBm, when used in an aircraft having a nominal 3 dB transmission line attenuation and mismatch loss and an antenna performance equivalent to that of a simple quarter-wave antenna. Other combinations of transponder peak pulse power output and MTL, transmission line loss and antenna performance, which result in comparable installed system effective radiated peak pulse power and MTL may be considered equally acceptable.

3.3.2 Antenna.

3.3.2.1 A technique which uses two transponders connected to separate antennas must be considered with extreme caution. Such an arrangement, unless carefully controlled, could result in unsatisfactory performance because of the difficulty of matching transponder parameters. This technique requires matching of the relevant characteristics specified in 2.7 and in particular, matching of the reply delay (2.7.11) to within 0.2 microsecond.

3.3.2.2 Any switching device that alternately changes the transponder from one antenna to another at a preset rate should be avoided. A preferred method, if dual antennas are used, is

through received signal amplitude comparison whereby the transponder reply is routed to the antenna which receives the stronger interrogation signal.

3.3.3 Transponder Self-Test and Monitor. If self-test and/or monitor devices are installed and used in aircraft to indicate normal or faulty operation, care should be exercised to minimize any system derogation (particularly fruit generation) that may result. The duration of use of the test mode should be an absolute minimum and limited to that required by the pilot to determine the transponder status. In order to minimize suppression of replies to ground interrogations, the test signal interrogation rate and level should be the lowest practicable for test.

3.3.4 Pressure-Altitude Transmission.

3.3.4.1 In order to achieve maximum operational benefit from automatic pressure-altitude transmission, the altitude information used by the pilot and that automatically provided to the controller must closely correspond (2.7.13.2.5). The highest degree of correspondence will be achieved by having airborne systems which use the same static pressure source, same aneroid unit, same static pressure error correction device and same scale correction device for both the pilot and the automatically transmitted pressure-altitude data.

3.3.4.2 For aircraft installations which are not yet equipped with altitude digitizer units, the use of Mode C reply framing pulses only (2.7.13.2.3) is encouraged as an interim arrangement.

3.3.4.3 The wording of the standard recognizes that facilities are provided on many transponders in service which only enable the information and framing pulses to be removed together. But its main objective is to ensure that inaccurate information pulses are removed while retaining the capability of position determination. The framing pulses alone are useful in certain ground processing equipments for enhancing the detection probability and azimuth accuracy.

3.3.4.4 The capability required by the Standard at 2.7.13.2.2 should be provided in new installations.

3.3.5 Automatic Conversion of Pressure Altitude Data to Indicated Altitude. Automatically transmitted pressure altitude data obtained via ATCRBS may be displayed to air traffic controllers directly after being decoded when such data indicates that the aircraft from which it is received is at or above the transition level. When the aircraft is below the transition level, such data could be misleading, since it is based upon the standard atmospheric pressure reference datum of 29.92 inches of mercury, while the pilot's altimeter is adjusted to a different reference. In this case, therefore, the data must be converted by application of an appropriate correction factor based upon the same reference datum as that to which the pilot's altimeter is set.

3.3.6 Transmission of the "X" Pulse. In 2.6.2, the position of the "X" pulse is specified as a technical standard to provide for possible future expansion of the system. It is recognized that though a majority of airborne transponders of later design contain an "X" pulse position, there are no means at present embodied to permit the operational use of this pulse. To do so, modifications of existing transponders and/or ancillary equipment would be necessary. The extent of modifications required would depend on the future function of the "X" pulse.

3.3.7 Transponder Low Sensitivity Setting. Many existing transponders are equipped with a low sensitivity setting (minus 12 dB below normal sensitivity) which is manually selectable by the pilot upon request of the controlling agency. This feature has been found useful as an interim technique for reducing transponder side lobe response. However, SLS is being implemented at interrogator sites and the low sensitivity feature will not be needed in new transponders.

23706

ALTITUDE TRANSMISSION CODE

UNIT DISTANCE REFLECTED BINARY CODE FOR 8 BITS

A4, B1, B2, B4, PULSES* →

D2, D4, A1, A2 PULSES*

	0000	0001	0011	0010	0110	0111	0101	0100	1100	1101	1111	1110	1010	1011	1001	1000
0000	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0001	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16
0011	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47
0010	63	62	61	60	59	58	57	56	55	54	53	52	51	50	49	48
0110	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79
0111	95	94	93	92	91	90	89	88	87	86	85	84	83	82	81	80
0101	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111
0100	127	126	125	124	123	122	121	120	119	118	117	116	115	114	113	112
1100	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143
1101	159	158	157	156	155	154	153	152	151	150	149	148	147	146	145	144
1111	160	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175
1110	191	190	189	188	187	186	185	184	183	182	181	180	179	178	177	176
1010	192	193	194	195	196	197	198	199	200	201	202	203	204	205	206	207
1011	223	222	221	220	219	218	217	216	215	214	213	212	211	210	209	208
1001	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239
1000	255	254	253	252	251	250	249	248	247	246	245	244	243	242	241	240

256 INCREMENTS (500 FOOT EACH) GIVING ALTITUDE FROM -1000 FEET TO 127,000 FEET

* 0 or 1 in a pulse position indicates the absence or presence of a pulse respectively

REPLY PULSE ASSIGNMENT

500 FOOT INCREMENT GRAY CODE

FOR 100 FOOT INCREMENT SEE BELOW

ALTITUDE IN THOUSANDS OF FEET

500 FOOT GRAY CODE FROM ABOVE

100 FOOT INCREMENT CODE

ALTITUDE IN FEET

100 FOOT INCREMENT TABLE		
100 FOOT PULSES	SUM OF DIGITS OF GRAY CODE	
C_1 C_2 C_4	ODD	EVEN
0 0 1	7	8
0 1 1	6	9
0 1 0	5	0
1 1 0	4	1
1 0 0	3	2

ALTITUDE TRANSMISSION CODE

TRANSPONDER REPLY PULSE ASSIGNMENT

AD-A037 225 INSTITUTE FOR DEFENSE ANALYSES ARLINGTON VA SCIENCE A--ETC F/G 17/7
A REVIEW AND ANALYSIS OF THE MITRE BEACON COLLISION AVOIDANCE S--ETC(U)
OCT 76 J J BAGNALL, L R DAUSIN, I W KAY DOT-FA74WA-3498

UNCLASSIFIED S-481 FAA-RD-77-2 NL

2 OF 3

AD A037225

APPENDIX B

PROBABILITY OF A DELAY IN ACQUISITION OF AN AIRCRAFT

Four consecutive interrogations must be successfully answered before the MITRE BCAS will initiate a track on an aircraft. The statistics for accomplishing this are developed in this appendix.

For ease of explanation, the following conventions will be adopted. The probability of receiving a reply from a particular aircraft during an epoch (i.e., round reliability) is p and q is equal to 1-p. The probability of acquisition with a delay of exactly x seconds is p(x) and the cumulative probability of acquisition with a delay of x seconds or less is P(x). In describing the acquisition process, the epochs will be referred to by number. The epoch during which the aircraft's range becomes less than the instrumented maximum range (i.e., 20 nmi) is the first epoch and other epochs are numbered consecutively from this one. The nth epoch is therefore the nth chance the equipment has to obtain a reply message from the aircraft. The delay in acquisition is equal to one less than the epoch number during which the first data sample is obtained as part of the required sequence of four replies. If four replies are received from the aircraft during the first through the fourth epochs, no delay is incurred.

Received data sample patterns are shown in Fig. B-1 for acquisition delays up to 7 seconds. In the figure, a √ indicates that a data sample is obtained during the indicated epoch

CASE	DELAY (sec)	EPOCH NUMBER											
		1	2	3	4	5	6	7	8	9	10	11	12
a	0	✓	✓	✓	✓								
b	1	X	✓	✓	✓	✓							
c	2	✓/X	X	✓	✓	✓	✓						
d	3	✓/X	✓/X	X	✓	✓	✓	✓					
e	4	✓/X	✓/X	✓/X	X	✓	✓	✓	✓				
f	5	✓/X	✓/X	✓/X	✓/X	X	✓	✓	✓	✓			
						BUT NOT							
		✓	✓	✓	✓	X	✓	✓	✓	✓			
g	6	✓/X	✓/X	✓/X	✓/X	✓/X	X	✓	✓	✓	✓		
						BUT NOT							
		✓	✓	✓	✓	✓/X	X	✓	✓	✓	✓		
						OR							
		X	✓	✓	✓	✓	X	✓	✓	✓	✓		
h	7	✓/X	✓/X	✓/X	✓/X	✓/X	✓/X	X	✓	✓	✓	✓	
						BUT NOT							
		✓	✓	✓	✓	✓/X	✓/X	X	✓	✓	✓	✓	
						OR							
		X	✓	✓	✓	✓	✓/X	X	✓	✓	✓	✓	
						OR							
		✓/X	X	✓	✓	✓	✓	X	✓	✓	✓	✓	

✓ INDICATES DATA ARE RECEIVED

X INDICATES DATA ARE NOT RECEIVED

✓/X INDICATES DATA CAN OR CANNOT BE RECEIVED

9-27-76-16

FIGURE B-1. Received Data Sample Patterns

number and an X indicates that the data are not obtained. Both a ✓ and an X indicates that it does not matter if the data sample is or is not obtained.

For acquisition delays of 0 through 4 seconds, only a single pattern is needed in order to show the required data sequence. For example, a delay of 0 requires four data samples (or hits) during epochs 1 through 4. Considering another example, the pattern for a delay of two seconds can have either a hit or a miss (i.e., no data sample in the epoch) in the first epoch, a miss in the second epoch and four hits during epochs 3 through 7.

The procedure for presenting the required data pattern for delays greater than 4 seconds is different than that described above in that a general pattern is first shown in which a number of epochs can have either hits or misses. This is followed by data patterns which must be excluded. For example, a delay of 5 seconds must have a miss followed by 4 hits in epochs 5 through 9 but can have either a hit or miss in epochs 1 through 4 except as noted. As shown in the figure, the pattern which is not allowed produces an acquisition earlier than the indicated delay (i.e., the excluded pattern would produce a delay of zero). For a delay of 6 seconds, the general pattern is shown with a miss and 4 hits in epochs 6 through 10 and either a hit or miss in the first 5 epochs. The patterns which must be excluded produce an acquisition delay of either 0 or 1 second. In more general terms, a delay of N seconds must have a miss followed by 4 hits in epochs N through N + 4 and epochs 1 through N-1 can contain either hits or misses as long as 4 hits in a row are not produced.

The mathematical formulation for the acquisition process is easy to derive from Fig. B-1. The probability of no delay is p^4 and the probability of a delay of 1, 2, 3 or 4 seconds is

$q\ p^4$. For greater delays, the probability is $q\ p^4$ times 1 minus the patterns which must be excluded. Mathematically, the probability for a delay of N seconds can be written as:

$$p(N) = q\ p^4(1-P(N-5)) \qquad \text{for } n \geq 5 \qquad \text{(B-1)}$$

A computer routine was written in order to evaluate the probability and cumulative probability of a delay in acquisition. The results are presented in Tables B-1 through B-10 for values of round reliability between 0.95 and 0.5. Also a listing of the computer routine is provided in Table B-11.

Acq. Delay (Sec)	Prob. of Acquisition	Cum. Prob. of Acquis.
0	0.815	0.815
1	0.041	0.855
2	0.041	0.896
3	0.041	0.937
4	0.041	0.977
5	0.008	0.985
6	0.006	0.991
7	0.004	0.995
8	0.003	0.998
9	0.001	0.999
10	0.001	0.999
11	0.000	1.000
12	0.000	1.000
13	0.000	1.000
14	0.000	1.000
15	0.000	1.000
16	0.000	1.000
17	0.000	1.000
18	0.000	1.000
19	0.000	1.000
20	0.000	1.000
21	0.000	1.000
22	0.000	1.000
23	0.000	1.000
24	0.000	1.000
25	0.000	1.000?

TABLE B-1. DELAY IN ACQUISITION FOR A ROUND RELIABILITY OF 0.95

Acq. Delay (Sec)	Prob. of Acquisition	Cum. Prob. of Acquis.
0	0.656	0.656
1	0.066	0.722
2	0.066	0.787
3	0.066	0.853
4	0.066	0.919
5	0.023	0.941
6	0.018	0.959
7	0.014	0.973
8	0.010	0.983
9	0.005	0.988
10	0.004	0.992
11	0.003	0.995
12	0.002	0.997
13	0.001	0.998
14	0.001	0.998
15	0.001	0.999
16	0.000	0.999
17	0.000	1.000
18	0.000	1.000
19	0.000	1.000
20	0.000	1.000
21	0.000	1.000
22	0.000	1.000
23	0.000	1.000
24	0.000	1.000
25	0.000	1.000

TABLE B-2. DELAY IN ACQUISITION FOR A ROUND RELIABILITY OF 0.9

Acq. Delay (Sec)	Prob. of Acquisition	Cum. Prob. of Acquis.
0	0.522	0.522
1	0.078	0.600
2	0.078	0.679
3	0.078	0.757
4	0.078	0.835
5	0.037	0.873
6	0.031	0.904
7	0.025	0.929
8	0.019	0.948
9	0.013	0.961
10	0.010	0.971
11	0.008	0.979
12	0.006	0.984
13	0.004	0.988
14	0.003	0.991
15	0.002	0.993
16	0.002	0.995
17	0.001	0.996
18	0.001	0.997
19	0.001	0.998
20	0.001	0.999
21	0.000	0.999
22	0.000	0.999
23	0.000	0.999
24	0.000	1.000
25	0.000	1.000

TABLE B-3. DELAY IN ACQUISITION FOR A ROUND RELIABILITY OF 0.85

Acq. Delay (Sec)	Prob. of Acquisition	Cum. Prob. of Acquis.
0	0.410	0.410
1	0.082	0.492
2	0.082	0.573
3	0.082	0.655
4	0.082	0.737
5	0.048	0.786
6	0.042	0.827
7	0.035	0.862
8	0.028	0.890
9	0.022	0.912
10	0.018	0.930
11	0.014	0.944
12	0.011	0.955
13	0.009	0.964
14	0.007	0.971
15	0.006	0.977
16	0.005	0.982
17	0.004	0.985
18	0.003	0.988
19	0.002	0.991
20	0.002	0.992
21	0.002	0.994
22	0.001	0.995
23	0.001	0.996
24	0.001	0.997
25	0.001	0.998

TABLE B-4. DELAY IN ACQUISITION FOR A ROUND RELIABILITY OF 0.8

Acq. Delay (Sec)	Prob. of Acquisition	Cum. Prob. of Acquis.
0	0.316	0.316
1	0.079	0.396
2	0.079	0.475
3	0.079	0.554
4	0.079	0.633
5	0.054	0.687
6	0.048	0.735
7	0.042	0.776
8	0.035	0.812
9	0.029	0.841
10	0.025	0.865
11	0.021	0.886
12	0.018	0.904
13	0.015	0.919
14	0.013	0.932
15	0.011	0.942
16	0.009	0.951
17	0.008	0.959
18	0.006	0.965
19	0.005	0.971
20	0.005	0.975
21	0.004	0.979
22	0.003	0.982
23	0.003	0.985
24	0.002	0.987
25	0.002	0.989

TABLE B-5. DELAY IN ACQUISITION FOR A ROUND RELIABILITY OF 0.75

Acq. Delay (Sec)	Prob. of Acquisition	Cum. Prob. of Acquis.
0	0.240	0.240
1	0.072	0.312
2	0.072	0.384
3	0.072	0.456
4	0.072	0.528
5	0.055	0.583
6	0.050	0.633
7	0.044	0.677
8	0.039	0.716
9	0.034	0.750
10	0.030	0.780
11	0.026	0.807
12	0.023	0.830
13	0.020	0.850
14	0.018	0.868
15	0.016	0.884
16	0.014	0.898
17	0.012	0.910
18	0.011	0.921
19	0.009	0.931
20	0.008	0.939
21	0.007	0.946
22	0.006	0.953
23	0.006	0.958
24	0.005	0.963
25	0.004	0.968

TABLE B-6. DELAY IN ACQUISITION FOR A ROUND RELIABILITY OF 0.7

Acq. Delay (Sec)	Prob. of Acquisition	Cum. Prob. of Acquis.
0	0.179	0.179
1	0.062	0.241
2	0.062	0.303
3	0.062	0.366
4	0.062	0.428
5	0.051	0.480
6	0.047	0.527
7	0.044	0.571
8	0.040	0.610
9	0.036	0.646
10	0.033	0.679
11	0.030	0.708
12	0.027	0.735
13	0.024	0.759
14	0.022	0.781
15	0.020	0.801
16	0.018	0.820
17	0.017	0.836
18	0.015	0.851
19	0.014	0.865
20	0.012	0.877
21	0.011	0.889
22	0.010	0.899
23	0.009	0.908
24	0.008	0.917
25	0.008	0.924

TABLE B-7. DELAY IN ACQUISITION FOR A ROUND RELIABILITY OF 0.65

Acq. Delay (Sec)	Prob. of Acquisition	Cum. Prob. of Acquis.
0	0.130	0.130
1	0.052	0.181
2	0.052	0.233
3	0.052	0.285
4	0.052	0.337
5	0.045	0.382
6	0.042	0.425
7	0.040	0.464
8	0.037	0.501
9	0.034	0.536
10	0.032	0.568
11	0.030	0.598
12	0.028	0.625
13	0.026	0.651
14	0.024	0.675
15	0.022	0.698
16	0.021	0.719
17	0.019	0.738
18	0.018	0.756
19	0.017	0.773
20	0.016	0.789
21	0.015	0.803
22	0.014	0.817
23	0.013	0.829
24	0.012	0.841
25	0.011	0.852

TABLE B-8. DELAY IN ACQUISITION FOR A ROUND RELIABILITY OF 0.6

Acq. Delay (Sec)	Prob. of Acquisition	Cum. Prob. of Acquis.
0	0.092	0.092
1	0.041	0.133
2	0.041	0.174
3	0.041	0.215
4	0.041	0.256
5	0.037	0.294
6	0.036	0.329
7	0.034	0.363
8	0.032	0.396
9	0.031	0.426
10	0.029	0.455
11	0.028	0.483
12	0.026	0.509
13	0.025	0.534
14	0.024	0.558
15	0.022	0.580
16	0.021	0.601
17	0.020	0.622
18	0.019	0.641
19	0.018	0.659
20	0.017	0.676
21	0.016	0.693
22	0.016	0.708
23	0.015	0.723
24	0.014	0.737
25	0.013	0.750

TABLE B-9. DELAY IN ACQUISITION FOR A ROUND RELIABILITY OF 0.55

Acq. Delay (Sec)	Prob. of Acquisition	Cum. Prob. of Acquis.
0	0.062	0.062
1	0.031	0.094
2	0.031	0.125
3	0.031	0.156
4	0.031	0.187
5	0.029	0.217
6	0.028	0.245
7	0.027	0.272
8	0.026	0.299
9	0.025	0.324
10	0.024	0.349
11	0.024	0.372
12	0.023	0.395
13	0.022	0.417
14	0.021	0.438
15	0.020	0.458
16	0.020	0.478
17	0.019	0.497
18	0.018	0.515
19	0.018	0.533
20	0.017	0.550
21	0.016	0.566
22	0.016	0.582
23	0.015	0.597
24	0.015	0.611
25	0.014	0.625

TABLE B-10. DELAY IN ACQUISITION FOR A ROUND RELIABILITY OF 0.5

```
10 DIMENSION A(26),B(26)
90 130 CONTINUE
100 INPUT,AP
110 AQ=1-AP
120 A(1)=AP**4.
121 B(1)=A(1)
130 A(2)=A(1)*AQ
131 A(3)=A(2)
132 A(5)=A(2)
133 A(4)=A(2)
140 B(2)=B(1)+A(2)
150 B(3)=B(2)+A(3)
160 B(4)=B(3)+A(4)
170 B(5)=B(4)+A(5)
180 DO 100 N=6,26
190 A(N)=A(2)*(1-B(N-5))
200 B(N)=B(N-1)+A(N)
210 100 CONTINUE
220 DO 110 M=1,26
230 PRINT 10 ,M-1,A(M),B(M)
240 10 FORMAT( I15,2F14.3)
250 110 CONTINUE
255 PRINT 12
256 12 FORMAT(//)
260 INPUT,AK
270 IF(AK .EQ. 0)GO TO 130
280 STOP;END
```

TABLE B-11. PROGRAM LISTING OF EVALUATING DELAY IN ACQUISITION

APPENDIX C

DETAILED ANALYSES OF GARBLE-GENERATED TRACKS

A. BRACKET DETECTIONS

The statistical characteristics of the garble pulses contributed by the overlapping replies provide a convenient starting point for the analysis of garble-generated tracks.

Each reply is 20.3 μsec long and consists of two bracket pulses plus an average of four additional altitude information pulses. The maximum number of altitude pulses is 11. Given N overlapping replies we get:

1. 6N/20.3 for the expected number of reply pulses per μsec of delay per interrogation;
2. 5N/20.3 for the expected pulse rate (per μsec of delay) for pulses which are not legitimate first pulses of bracket pairs; and
3. 5N/20.3 for the expected pulse rate (per μsec of delay) for pulses which are not legitimate second pulses of bracket pairs.

Given any one reply pulse, which is not a legitimate first pulse of a bracket pair, the probability of finding a second pulse within 20.3 ± 0.25 μsec (=window where second bracket pulses are detected) is evaluated as follows:

a. Let z_n denote the number of pulses in one of the N overlapping replies, i.e., $1 \le n \le N$, which could contribute an illegitimate second bracket pulse;
b. z_n-1 is the number of pulses in the nth reply exclusive of the legitimate second bracket pulse;

c. $(z_n-1) \times 0.5/20.3$ is the probability that one of the pulses in (b) will contribute an illegitimate second bracket pulse;

so that the probability that none of the N overlapping replies contribute an illegitimate second bracket pulse is the N-fold product

$$\prod_{n=1}^{N} \left[1 - \frac{(z_n-1)x0.5}{20.3}\right] .$$

This, in fact, is a conditional probability, the condition being that the overlapping replies contain $z_1, \ldots, z_N$ pulses. To obtain the unconditional probability, the above expression must be averaged over the z_n. Since the number of pulses, z_n, in any one reply is independent from that in another reply, the factors can be averaged one at a time, and each gives

$$1 - \frac{(\bar{z}-1)0.5}{20.3} = 1 - \frac{5x0.5}{20.3} = 0.877 \qquad \text{(C-1)}$$

where $\bar{z} = 6$ is the average number of pulses per reply. Thus, the N-fold product becomes $(0.877)^N$ and the probability that one or more of the overlapping replies contribute an illegitimate second bracket pulse is

$$1 - (0.877)^N .$$

Multiplication by the expected rate of illegitimate first bracket pulses (see Item 2 at the start of this section) gives:

$$\text{Expected rate of false bracket detections, per } \mu\text{sec of delay, per interrogation} = \frac{5N}{20.3}[1 - (0.877)^N] . \qquad \text{(C-2)}$$

The expected number of false brackets formed from all replies received from aircraft out to 20 nmi is obtained by multiplying Eq. C-2 by 247 μsec (≈round trip delay to 20 nmi). Consequently,

$$\text{Expected number of false brackets per interrogation} = 247 \times \frac{5N}{20.3}\left[1-(0.877)^N\right]. \quad \text{(C-3)}$$

After a second interrogation, one second later, the expected number of false bracket detections, whose apparent range has closed by 0 to 1650 ft (≈977 knots) relative to a given false bracket from the previous interrogation is obtained by multiplying Eq. C-2 by 3.354 μsec (corresponding to a 1650 ft range interval). This result, when multiplied by Eq. C-2 yields the expected number of false bracket pairs, per interrogation, meeting the closing range criterion:

$$\text{Expected number of false bracket pairs per interrogation} = 3.354 \times 247 \times \left(\frac{5N}{20.3}\right)^2 \left[1-(0.877)^N\right]^2. \quad \text{(C-4)}$$

where the pair is formed from two successive interrogations.

B. TRACK ACQUISITION

The range and range difference between bracket pairs (obtained on successive interrogations) is used by MCAS to extrapolate the apparent aircraft range to an earlier and a later interrogation (cf. Fig. 4 and related discussion in Section III-E-1). At each extrapolated range, a range window of ±240 ft is examined for the presence of a bracket pulse pair. The expected number of false brackets inside either window is the product of 0.976 μsec (corresponding to the 480 ft window) and Eq. C-2:

$$\text{Expected number of false brackets in extrapolated window} = 0.976 \times \frac{5N}{20.3}\left[1-(0.877)^N\right]. \quad \text{(C-5)}$$

Thus, the expected number of false bracket quadruplets (see Cf. Section III-E-1) is the product of Eq. C-4 and the square of Eq. C-5:

$$\begin{array}{l}\text{Expected number of false}\\ \text{bracket quadruplets gen-}\\ \text{erated on each interrogation}\end{array} = 2.98\, N^4 \left[1-(0.877)^N\right]^4 . \qquad \text{(C-6)}$$

Each of the false brackets of the quadruplet will contain garble detections at the nominal altitude pulse positions. If the "anded" detections produce a legitimate altitude code, a track will be initiated. The illegitimate altitude codes are those whose C-bits (cf. Fig. 2, Section III-D) form the binary sequences: 000, 101, and 111. The probability that the "anded" garble detections, at any given altitude pulse position, produce a binary "one" is the probability of a garble pulse in each of the four brackets at the given altitude pulse position. This probability is denoted by p, so that the probability of a binary "zero" is 1-p, and the probabilities of obtaining each of the illegal altitude codes are:

$$\begin{aligned} Pr[000] &= (1-p)^3 \\ Pr[101] &= p^2(1-p) \qquad \text{(C-7a, b, c)} \\ Pr[111] &= p^3 \end{aligned}$$

and

$$\begin{array}{l}\text{Probability that "anded" garble}\\ \text{produces a legitimate C-bit}\\ \text{sequence}\end{array} \begin{aligned} &= 1 - (1-p)^3 - p^2(1-p) - p^3 \\ &= 1 - p^2 - (1-p)^3 . \qquad \text{(C-8)}\end{aligned}$$

The probability, p, of obtaining garble pulses in a given altitude pulse position on four successive interrogations, is

$$p = \Pr[\text{garble \#1}] \times \Pr[\text{garble \#2}|\text{garble \#1}] \times \Pr[\text{garble \#3}|\text{garbles \#2 \& 1}] \times \Pr[\text{garble \#4}|\text{garbles 3, 2, \& 1}], \qquad \text{(C-9)}$$

where the notation within the brackets denotes the garble events for the four interrogations, and the vertical line denotes the conditional probabilities. The dependence between events is due to garble contributors whose range rate matches, approximately, the apparent range rate of the false bracket quadruplet. The approximate range rate match is obtained when the magnitude of the range rate difference is less than one-half the pulse width (=0.45 μsec x 492 ft/μsec = 221.4 ft) divided by 1 sec, the time between interrogations; in other words, when the range rate difference is less than 111 ft/sec.

The conditional probability that the range rate difference is within ±111 ft/sec, given a replying aircraft range rate v, is the probability that the apparent range rate of the false bracket quadruplet lies between v-111 ft/sec and v+111 ft/sec. Since the apparent range rates are uniformly distributed between 0 and 1650 ft/sec (see the bracket formation criteria discussed between Eqs. C-3 and C-4), the conditional probability, as a function of v, is as shown in Fig. C-1.

The probability of a range rate match, i.e., the probability that the range rate difference is less than ±111 ft/sec, is

$$\Pr[\text{match}] = \int_{-111}^{1761} \Pr[\text{match}|v]\, f(v)dv, \qquad \text{(C-10)}$$

where Pr[match|v] is the conditional probability of a match, given that the garble contributor range rate is v; this conditional probability is the plot shown in Fig. C-1; and f(v) is

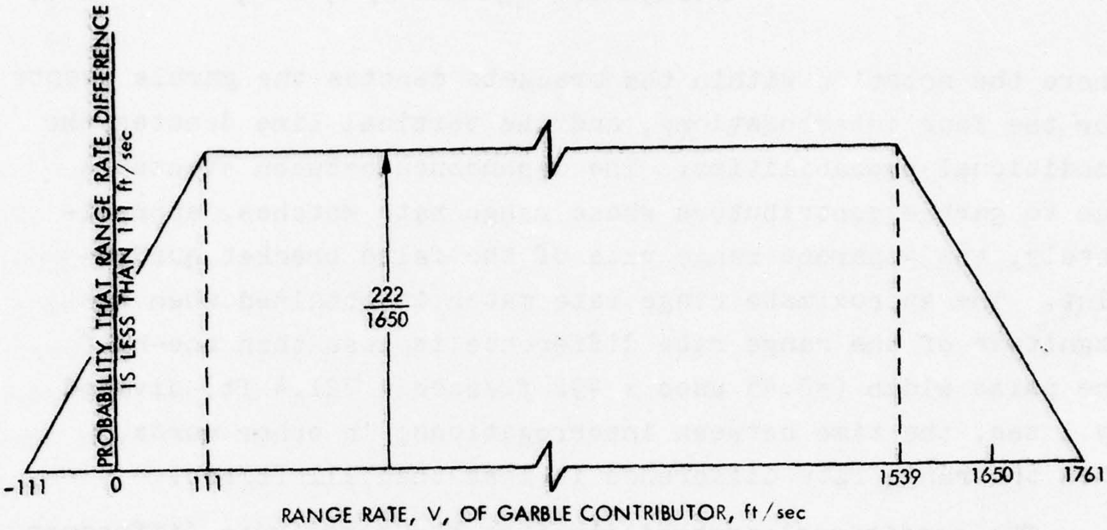


FIGURE C-1. Conditional Probability that the Range Rate Difference Between a Garble Contributor and the False Bracket Quadruplet is Within ±111 ft/sec, as a Function of the Range Rate of the Garble Contributor.

the distribution density function between the interrogator and responder population. Thus, using Fig. C-1 in Eq. C-10 we get

$$\Pr[\text{match}] = \int_{-111}^{111} \left[\frac{1}{2}\,\frac{222}{1650} + \frac{1}{1650}\,v\right] f(v)dv$$

$$+ \frac{222}{1650}\int_{111}^{1539} f(v)dv, \qquad \text{(C-11)}$$

where f(v) is assumed to make no significant contribution beyond 1539 ft/sec. Furthermore, f(v) is symmetric about v = 0 (for every approaching aircraft there is a receding aircraft) so that:

1. The integral of the antisymmetric function vf(v) from -111 to +111 goes to zero.
2. The integral of f(v) from -111 to +111 is twice the integral from 0 to +111.

Under these conditions, Eq. C-11 reduces to

$$\Pr[\text{match}] = \frac{222}{1650}\int_{0}^{1539} f(v)dv$$

$$= \frac{222}{1650} \bullet \frac{1}{2} = 0.0673 = S, \qquad \text{(C-12)}$$

where the integral is $\frac{1}{2}$ because, for all practical purposes, half of the responding aircraft will have closing range rates between 0 and 1650 ft/sec.

The conditional probability of detecting a garble pulse (at a given position within the false bracket), given that one was detected on a previous interrogation, is evaluated by considering the two possibilities: (I) none of the previous garble contributors were matched, in which case the probability of garble on the present interrogation is $1 - A^N$ where A = probability of no garble from any one reply, and A^N is the probability that none of the N overlapping replies contribute

garble; (II) one or more of the previous garble contributors were matched, in which case the probability of garble on the present interrogation is unity. However, the probability that none of the previous garble contributors were matched, i.e., Case I, is $(1-S)^N$ (S is given by Eq. C-12), and the probability that at least one of them is matched, i.e., Case II, is $1 - (1-S)^N$. Thus, the probability of a garble detection (at a given position within the bracket) on the present interrogation, given a garble detection on the previous interrogation becomes,

$$(1-A^N)(1-S)^N + 1 \bullet [1 - (1-S)^N] = 1 - [A(1-S)]^N. \quad \text{(C-13)}$$

This represents the second factor in the chain of probabilities in Eq. C-9. If the probability that a given garble contributor remains matched for three interrogations is neglected*, then Eq. C-13 can also be used for the third and fourth factors of Eq. C-9; while the first factor is $1 - A^N$. Under these conditions, Eq. C-9 becomes

$$p = (1-A^N) \bullet [1 - A^N(1-S)^N]^3 . \quad \text{(C-14)}$$

The conditional probability, A_z, that a given reply, containing z pulses, will not contribute garble at a given time within 20.3 μsec is

$$A_z = \frac{20.3\ \mu\text{sec} - z \times 0.45\ \mu\text{sec}}{20.3\ \mu\text{sec}} , \quad \text{(C-15)}$$

*By repeating the analysis which led up to Eq. C-12, this probability can be shown to be 3.4 percent.

in which 0.45 μsec is the width of each pulse.* The unconditional probability, A, is obtained by averaging over z:

$$A = 1 - \frac{\bar{z} \times 0.45}{20.3} = 1 - \frac{6 \times 0.45}{20.3} = 0.867, \qquad \text{(C-16)}$$

where $\bar{z} = 6$ is the average number of pulses in a reply. Substituting Eqs. C-12 and C-16 into Eq. C-14, we obtain

$$p = [1 - (0.867)^N][1 - (0.809)^N]^3 . \qquad \text{(C-17)}$$

To sum up: Eq. C-17 gives the probability, p, that the "anded" garble produces a binary "one", while Eq. C-8 gives the probability that the resultant sequence is a legitimate C-bit code. Furthermore, the probability of obtaining at least one binary "one" among the A/B bits** (i.e., A_1, A_2, A_4, B_1, B_2, B_4 for altitudes below 31,000 ft as

$$\text{Probability of at least one "one" among A/B bits} = 1 - (1-p)^6 \qquad \text{(C-17à)}$$

Since a track acquisition is produced by every bracket quadruplet whose "anded" data produce a legitimate C-bit sequence and at least one "one" among the A/B bits, the product of Eqs. C-6, C-8, and C-17a gives the expected rate of false track acquisitions. Sample calculations are summarized in Table C-1.

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*This analysis assumes that detections at a given altitude pulse position are derived from a single tap of the 168-bit shift register of the bracket detector. However, the MITRE BCAS detects altitude pulses from the logical "or" of two adjacent registers, 0.121 μsec (= one clock period) apart. The effect of this is that the 0.45 μsec value, used in Eq. C-15, is replaced by 0.45 + 0.121 = 0.571 μsec. When the subsequent analysis (leading up to Tables C-9, C-10, C-11) is repeated, for N = 4, it is found that the false alarm rate is increased by a factor of 17 while the average track load is increased from 206 to a value above 340.

**The highest altitude bit among A_1, A_2, A_4, B_1, B_2, B4 is A_1 which is used between 15,000 ft and 31,000 ft. Thus, the 6-bit sequence is the minimum needed to cover the prevalent aircraft populations. (cf. Fig. 2, Section III-D).

TABLE C-1. COMPUTATION OF THE FALSE TRACK ACQUISITION RATE

Statistic	Number of overlapping replies N=3	N=4	N=5	Method of computation
(A) Expected number of false bracket quadruplets per second	2.71	21.2	99.9	Eq. C-6
(B) Probability that the "anded" garble within the bracket quadruplet produces a legitimate C-bit sequence	0.104	0.218	0.349	Eqs. C-8 and C-17
(C) Probability that the "anded" garble within the bracket quadruplet produces at least one "one" among the A/B bits	0.199	0.399	0.602	Eqs. C-17 and C-17a
(D) Expected number of false track acquisitions per second	0.0561	1.84	21.0	Product of above

C. REPLY PROCESSING AFTER TRACK ACQUISITION

After track acquisition, the predicted range window (±240 ft centered on the extrapolated range) is examined for bracket pulse pairs. If a bracket is detected, and if the bracketted pulses meet the implemented logic criteria (to be discussed later in this section), then the track is updated. Given a false track initiation, the expected number of brackets detected in the projected window is a sum of two contributions:

1. False bracket detections represented by the false bracket detection rate given by Eq. C-2, plus
2. Detected legitimate brackets for replies whose range falls within ±240 ft of the false track range for the given interrogation. Such replies, if they meet the subsequent logic criteria, would be used to update both the legitimate track and/or the false track, or neither track. The expected rate of legitimate brackets, per μsec of delay is N/20.3 μsec.

The sum of (1) and (2), multiplied by 0.976 μsec (corresponding to the 480 ft window) gives

$$\text{Expected number of bracket detections per range window, per interrogation} = 0.976\left\{\frac{5N}{20.3}\left[1-(0.877)^{N}\right] + \frac{N}{20.3}\right\}, \quad \text{(C-18)}$$

The data detected within a bracket will be rejected if either one or both of the following conditions are met:

$$\text{RPGRY} \cap 7 = 0, \quad \text{(C-19)}$$

and/or

$$\text{RPGRY} \cap \text{PGRAY} \cap 7 = 0, \quad \text{(C-20)}$$

where RPGRY represents the three C-bits detected in the altitude code; the numeral 7 represents the sequence of three binary

"ones"; $\cap$ represents the logical "and", or "intersection", of the two binary sequences; the numeral 0 represents the binary sequence of three "zeros"; PGRAY represents the C-bits of the three predicted altitudes: one nominal altitude, or "best" prediction and two adjacent altitudes within ±100 ft of the "best" value. If the data pass the tests embodied in Eqs. C-19 and C-20 then a subsequent correlation number test is applied. This test is implemented by evaluating

$$\text{correlation number} = 48 - 3M - Q \ , \qquad \text{(C-21)}$$

where M represents the number of binary "ones" obtained from

$$RPGRY \cap \overline{PGRAY},$$

(the dashed line denotes the logical complement) and Q is the number of binary "ones" in

$$\overline{RPGRY} \cap PGRAY,$$

for the complete set of altitude bits (C-bits as well as others). In effect, M is the number of unexpected "ones", i.e., "ones" appearing in the detected bit sequence where the prediction has "zeros"; while Q is the number of unexpected "zeros", i.e., "zeros" where the prediction has "ones". If the detected bit sequence passes the tests represented by Eqs. C-19 and C-20, and if the correlation number exceeds a minimum threshold for one or more of the three predicted altitudes, then the detected sequence is accepted as a legitimate reply. MITRE has experimented (by computer simulation) with threshold settings from 36 to 44 and no final choice has yet been made.

It is clear that as the threshold is increased, garble reply rejection improves, but the probability that a legitimate reply is accepted deteriorates. Thus, any choice of threshold represents a compromise between these performance characteristics. In order to gain some insight into the problem, the average

correlation number for garble alone, and for garble plus a legitimate reply, will be evaluated in subsequent analyses. For this purpose it is convenient to use Eq. C-21 in several alternative forms:

$$\begin{aligned} \text{correlation number} &= 48 - 3M - Q \\ &= 48 - 3\,(M+\tfrac{1}{3}\,Q) \\ &= 48 - 3\,[M_c+M_x+\tfrac{1}{3}\,Q_c+\tfrac{1}{3}\,Q_x], \end{aligned} \tag{C-22}$$

where M_c represents the contribution to M from the C-bits, while M_x represents the contribution of other bits; and similarly, Q_c and Q_x represent the contributions to Q from the C-bits and from the other bits. Furthermore, it is convenient to define decorrelation numbers D, D_c, and D_x by

$$D = M + \frac{1}{3}\,Q\ , \tag{C-23}$$

$$D_x = M_x + \frac{1}{3}\,Q_x\ , \tag{C-24}$$

$$D_c = M_c + \frac{1}{3}\,Q_c\ , \tag{C-25}$$

so that

$$D = D_c + D_x\ , \tag{C-26}$$

and Eq. 22 becomes

$$\begin{aligned} \text{correlation number} &= 48 - 3D \\ &= 48 - 3D_c - 3D_x\ . \end{aligned} \tag{C-27}$$

The values of M_c, Q_c, and D_c obtained for different combinations of predicted and received sequences are enumerated in Table C-3.

The C-bit sequences that can appear in the three predicted altitudes ("best" plus two adjacent altitudes) are enumerated across the top row of Table C-4 and are designated as predicted triplets TR1, ..., TR5. The left-hand column of Table C-4 lists, again, all the possible combinations of "ones" and "zeros" that can appear in the presumed C-bit positions. The entries in the

TABLE C-3. COMPUTATION OF THE DECORRELATION NUMBER FOR GARBLE C-BIT SEQUENCES

Detected Garble Sequence	Predicted C-bit Sequences 001 M_c	001 Q_c	001 D_c	011 M_c	011 Q_c	011 D_c	010 M_c	010 Q_c	010 D_c	110 M_c	110 Q_c	110 D_c	100 M_c	100 Q_c	100 D_c
111	2	0	2	1	0	1	2	0	2	0	1	1/3	2	0	2
011	1	0	1	0	0	0	1	0	1	1	1	4/3	Rejected(1)		
101	1	0	1	1	1	4/3	Rejected(1)			1	1	4/3	1	0	1
110	Rejected(1)			1	1	4/3	1	0	1	0	0	0	1	0	1
100	Rejected(1)			Rejected(1)			Rejected(1)			0	1	1/3	0	0	0
010	Rejected(1)			0	1	1/3	0	0	0	0	1	1/3	Rejected(1)		
001	0	0	0	0	1	1/3	Rejected(1)			Rejected(1)			Rejected(1)		
000	Rejected(2)			Rejected(2)			Rejected(2)			Rejected(2)			Rejected(2)		

(1) In accordance with Eq. C-20.
(2) In accordance with Eq. C-19.

TABLE C-4. MINIMUM DECORRELATION NUMBERS, D_c, BETWEEN THE C-BITS OF THE PREDICTED TRIPLET AND DETECTED GARBLE SEQUENCES

Detected Garble Sequences	Predicted Altitude Triplets TR1 001 001 011	TR2 001 011 010	TR3 011 010 110	TR4 010 110 100	TR5 110 100 100
(a) 111	1	1	1/3	1/3	1/3
(b) 011	0	0	0	1	4/3
(c) 101	1	1	4/3	1	1
(d) 110	4/3	1	0	0	0
(e) 100	Rejected(1)	Rejected(1)	1/3	0	0
(f) 010	1/3	0	0	0	1/3
(g) 001	0	0	1/3	Rejected(1)	Rejected(1)
(h) 000	← Rejected(2) →				

(1) In accordance with Eq. C-20.
(2) In accordance with Eq. C-19.

table show the smallest of the three decorrelation numbers (corresponding to the largest correlation number) obtained between the detected garble and the three predictions. In addition, a rejection is shown if the detected sequence satisfies Eqs. C-19 and/or Eq. C-20 for each of the three predictions.

The distribution of the garble sequences, listed in the left-hand column, follow the Binomial distribution with the probability of a garble "one" given by

$$g = 1 - A^N = 1 - (0.867)^N \qquad \text{(C-28)}$$

where A is the probability that an overlapping reply does not contribute garble (see Eq. C-16). Thus, the probability that the garble sequence is rejected prior to correlation is the probability of occurrence of the "rejected" sequence in Table C-4, i.e.,

$g(1-g)^2 + (1-g)^3$ for triplets TR1, TR2, TR4, TR5

$(1-g)^3$ for triplet TR3.

Consequently the conditional probabilities of a garble C-bit sequence, given that it was not rejected prior to correlation, and given a specific predicted triplet, are

$$\frac{g^k\,(1-g)^{3-k}}{1-g(1-g)^2-(1-g)^3} \quad \text{for TR1, TR2, TR4, TR5}$$

and

$$\frac{g^k(1-g)^{3-k}}{1-(1-g)^3} \quad \text{for TR3}$$

where k is the number of garble "ones" in the garble C-bit sequence. The average value of the minimum decorrelation coefficient D_c (tabulated in Table C-4) is obtained by weighting the entries in Table C-4 by the above conditional probabilities. The results denoted by $\overline{D}_{c1}, \ldots, \overline{D}_{c5}$, for each of the five triplets, are:

$$\overline{D}_{c1} = \frac{g^3 + \frac{7}{3} g^2(1-g) + \frac{1}{3} g(1-g)^2}{1 - g(1-g)^2 - (1-g)^3} \quad \text{(C-29a)}$$

$$\overline{D}_{c2} = \frac{g^3 + 2g^2(1-g)}{1 - g(1-g)^2 - (1-g)^3} \quad \text{(C-29b)}$$

$$\overline{D}_{c3} = \frac{\frac{1}{3} g^3 + \frac{4}{3} g^2(1-g) + \frac{2}{3} g(1-g)^2}{1 - (1-g)^3} \quad \text{(C-29c)}$$

$$\overline{D}_{c4} = \frac{\frac{1}{3} g^3 + 2\, g^2(1-g)}{1 - g(1-g)^2 - (1-g)^3} \quad \text{(C-29d)}$$

$$\overline{D}_{c5} = \frac{\frac{1}{3} g^3 + \frac{7}{3} g^2(1-g) + \frac{1}{3} g(1-g)^2}{1 - g(1-g)^2 - (1-g)^3} \quad \text{(C-29e)}$$

The probability that any one of the triplets is used in the prediction is the probability that the "anded" garble at track initiation produces the binary sequence appearing in the middle of each of the triplets listed across the top row of Table C-4. This middle sequence is the "best" prediction while the other two are part of the "adjacent" sequences (within ±100 ft of the "best" prediction). The probability of occurrence of any one of the "best" prediction sequences is governed by:

1. The probability, p, given by Eq. C-17, that the "anded" garble produces a binary "one" in any one bit position.
2. The condition that the "illegal" C-bit sequences have been removed at track initiation (see row B of Table C-1).

Consequently,

$$\begin{array}{l}\text{Conditional probability of a} \\ \text{given legal C-bit sequence}\end{array} = \frac{p^k (1-p)^{3-k}}{1-p^2 - (1-p)^3} \qquad \text{(C-30)}$$

where k is the number of "ones" in the "best" predicted sequence, 3-k is the number of "zeros", and the denominator is the unconditioned probability of obtaining any one of the legal sequences (see Eq. C-8). Equation C-30, when summed over all the "legal" C-bit sequences:

001 (k=1)
011 (k=2)
010 (k=1)
110 (k=2)
100 (k=1)

gives unity probability.

Using the conditional average decorrelation (Eqs. C-29), given a particular triplet, and the probability of that triplet, Eq. C-30, we obtain the average decorrelation number, as shown in Table C-5, contributed by the C-bit positions.

Contributions from the six A/B bits are expressed in terms of the number, x, of "ones" in the predicted A/B bits. Thus, the corresponding number of zeros is 6-x. Since g, given by Eq. C-28, is the probability of a garble "one" after track initiation, the average number of unexpected "ones" (as defined below Eq. C-21) is (6-x)g and the average number of unexpected zeros is x(1-g). Both of these averages are conditioned on x so that the unconditional averages are

$$\overline{M}_x = (6-\overline{x})g \qquad \text{(C-31a)}$$

and

$$\overline{Q}_x = \overline{x}(1-g) \qquad \text{(C-31b)}$$

where $\overline{M}_x$ and $\overline{Q}_x$ is the average number of unexpected "ones" and "zeros" respectively.

TABLE C-5. COMPUTATION OF THE AVERAGE DECORRELATION, D_c, CONTRIBUTED BY THE C-BIT GARBLE (WITHOUT A LEGITIMATE TRACK)

Predicted Triplet	Conditional Average[1] Decorrelation, Given The Indicated Triplet		Probability[2] of Indicated Triplet	
	N=4	N=5	N=4	N=5
TR1	0.5553	0.6197	0.3148	0.3001
TR2	0.4350	0.5101	0.0278	0.0498
TR3	0.3203	0.3352	0.3148	0.3001
TR4	0.3544	0.3937	0.0278	0.0498
TR5	0.4747	0.5033	0.3148	0.3001

Average[3] decorrelation $\overline{D}_c = \begin{cases} 0.4644 \text{ for } N=4 \\ 0.4826 \text{ for } N=5 \end{cases}$

[1] Computed from Eq. C-29.
[2] Computed from Eq. C-30.
[3] Sum of products of the conditional average and the corresponding probabilities.

Since tracks are initiated only in those cases where $x \geq 1$, i.e., cases where at least one garble "one" is detected among the six A/B bits (see row C of Table C-1 and associated discussion below Eq. C-17) we have

$$\overline{x} = 1 + (6-1)p = 1 + 5p \quad \text{(C-32)}$$

where p, given by Eq. C-17, is the probability that the "anded" garble at track initiation produces a garble "one". Equation 32 is substituted into Eqs. C-31 which are then used to obtain

$$\begin{aligned} \overline{D}_x &= M_x + \frac{1}{3}\overline{Q}_x \\ &= 5(1-p)g + \frac{1}{3}(1+5p)(1-g) \quad \text{(C-33)} \\ &= \begin{cases} 2.26 & \text{for } N=4 \\ 2.47 & \text{for } N=5 \end{cases} \end{aligned}$$

in which $\overline{D}_x$ is the average value of D_x defined by Eq. C-24, and where the last step was obtained by using Eqs. C-17 and C-28 for p and g, respectively.

Reference to Eq. C-26 shows that the average decorrelation is the sum of the averages computed in Table C-5 and in Eq. C-33:

$$\text{Average decorrelation for garble alone} = \overline{D}_c + \overline{D}_x = \begin{cases} 2.73 & \text{for } N=4 \\ 2.95 & \text{for } N=5 \end{cases} \qquad \text{(C-34)}$$

The correlation number for a legitimate reply is computed from the following example:

1. Assume that a legitimate altitude code has two binary "ones": one C-bit and one A or B bit.
2. Assume that the best predicted altitude is an exact replica of the transmitted reply which, in the absence of garble, would yield the maximum correlation number of 48.
3. Assume that all transmitted "ones" are detected so that Q = 0 (see discussion below Eq. C-21).
4. The total number of bit positions among the A, B, and C bits (the same that were used earlier in evaluating the decorrelation for garble alone) is nine which, after subtracting the two legitimate bits in (1) leaves seven opportunities for receiving garble "ones".

Thus, the average number of unexpected "ones" is seven times the probability of a garble one (after track initiation). The latter is of the form of Eq. C-28 with N replaced by N-1 to account for the fact that one of the N overlapping replies is the le timate reply which leaves only N-1 garble contributors. Since the number of unexpected "zeros" is zero, by virtue of the assumption in item (3) above, we get

$$\text{Average decorrelation with legitimate reply} = 7\left[1 - (0.867)^{N-1}\right] \qquad \text{(C-35a)}$$

$$= \begin{cases} 2.44 & \text{for } N=4 \\ 3.04 & \text{for } N=5 \end{cases} \qquad \text{(C-35b)}$$

The correlation number, which is equal to 48 minus three times the decorrelation (see Eq. C-27), is shown in Table C-6.

Thus, to ensure that a lightweight legitimate reply (i.e., a reply containing only two "ones") is not rejected, the correlation threshold would have to be set below 39, in which case, an all-garble reply would also be accepted by the correlation logic. Thus, as far as rejection of all-garble replies is concerned, the correlation number logic is redundant: all of the other data processing logic has already eliminated the all-garble replies whose correlation number would have been well below that of a lightweight legitimate reply.

Specifically, the other logic will eliminate:

(1) The "illegal" C-bit sequences at track initiation (see analysis leading up to Eq. C-7).

(2) The rejectable C-bit sequences shown in Table C-4; and these include C-bit sequences which have no binary "one" in common with the prediction.

(3) All-garble replies without any "ones" among the A/B bits (see row C of Table C-1, Eq. C-17a and associated discussion).

(4) All-garble replies without any "ones" in common with the predicted A/B bits.

TABLE C-6. COMPARISON OF AVERAGE CORRELATION NUMBER

Type of Reply	Average Correlation[1] for N=4	N=5
Legitimate reply[2] plus garble	40.7	38.9
Garble only	39.8	39.2

[1] Forty-eight minus three times the decorrelation given by Eq. C-34 and Eq. C-35b.

[2] Legitimate reply contains only one C-bit "one" and one A or B bit "one".

If x denotes the number of A/B-bit "ones" in the best prediction then the number of A/B bit "ones" in the two adjacent predictions will be

$$x \text{ when triplets TR2, TR3, and TR4 are used}$$

and

$$x+1 \text{ when triplets TR1 and TR5 are used.}$$

These triplets are identified across the top row of Table C-4 which shows that two C-bit sequences in each of the triplets, TR1 and TR5, are identical; this means that one of the adjacent predictions contains an additional binary "one" among its A/B bits.

Thus, the probability that an all-garble reply has at least one A/B-bit "one" in common with the prediction is

$$1 - (1-g)^x \text{ for triplets TR2, TR3, and TR4}$$

and

$$1 - (1-g)^{x+1} \text{ for triplets TR1 and TR5}$$

where g, given by Eq. C-28, is the probability of a garble "one", and x is the number of "ones" among the A/B bits of the best predicted altitude. Because tracks are not initiated without any A/B-bit "ones" (see row C of Table C-1, Eq. C-17a and associated discussion), the value of x is always greater than or equal to one so that an upper bound on the above probabilities is given by

$$\begin{array}{l}\text{Probability of one}\\ \text{or more A/B garble}\\ \text{"ones" in common with}\\ \text{the predictions}\end{array} \geq \left\{\begin{array}{l} g \text{ for triplets TR2, TR3, TR4} \\ \\ 1-(1-g)^2 \text{ for triplets TR1, TR5} \end{array}\right. \qquad \text{(C-36)}$$

Furthermore, the probability that an all-garble reply has an acceptable C-bit sequence, i.e., one which is not rejected by the logic embodied in Eq. C-19 and Eq. C-20, is given by one minus the probability of the rejectable garble sequences identified in Table C-4. In other words

$$\text{Probability of an acceptable C-bit garble sequence} = \begin{cases} 1-g(1-g)^2 - (1-g)^3 = 1-(1-g)^2 \text{ for TR1, TR2, TR4, TR5} \\ 1-(1-g)^3 \text{ for TR3} \end{cases} \quad \text{(C-37)}$$

where $g(1-g)^2$ is the probability of the garble sequence (e) and also of the garble sequence (g) identified in Table C-4, and $(1-g)^3$ is the probability of sequence (h).

The product of Eq. C-36 and Eq. C-37 is the joint probability of the two events:

(1) An acceptable C-bit sequence, i.e., one which is not one of the rejectable sequences identified in Table C-4. This also guarantees that there is at least one binary "one" in common with the prediction.

(2) Detection of at least one binary "one" in common with the predicted A/B bits. This condition, in addition to a bracket detection and an acceptable C-bit sequence, insures that the track is extended either as a branch (or so-called "child") track or simply as the initial track itself.

Accordingly, the product of (1) and (2) is the probability that an all-garble altitude data sequence is accepted as a legitimate altitude code. Numerical results are shown in Table C-7 which are actually lower bounds because of the inequality of Eq. C-36.

Since a reply consists of bracket and alititude detections, α_N from Table C-7, multiplied by Eq. C-18 gives

$$\text{Expected number, } \phi\text{, of false replies per interrogation, per range window} = \phi = 0.976\alpha_N \left\{ \frac{5N}{20.3} [1 - (0.877)^N] + \frac{N}{20.3} \right\} \quad \text{(C-38)}$$

TABLE C-7. PROBABILITY[1] THAT AN ALL-GARBLE DATA SEQUENCE IS ACCEPTED AS LEGITIMATE ALTITUDE CODE

Predicted Triplet	Number of Overlapping Replies, N, N=3	N=4	N=5
TR1	0.3309	0.4634	0.5776
TR2	0.2004	0.2961	0.3877
TR3	0.2519	0.3565	0.4501
TR4	0.2004	0.2961	0.3877
TR5	0.3309	0.4634	0.5776
Overall probability[2], α_N	0.3020	0.4204	0.5204

(1) Probabilities for a given triplet are obtained as the product of Eq. C-36 and Eq. C-37.

(2) This is an overall average where the probability of occurrence of each triplet is taken from Eq. C-30, in which p is given by Eq. C-17.

The relationship between the mean value, ϕ, and the probability of at least one reply, is assumed to be that obtaining for a Poisson distribution:

$$\text{Probability, F, of at least one false reply per interrogation per range window} = F = 1-e^{-\phi}. \qquad \text{(C-39)}$$

Equations C-38 and C-39 together with α_N from Table C-7, are used to obtain

$$F = \begin{cases} 0.1081 \text{ for } N=3 \\ 0.2180 \text{ for } N=4 \\ 0.3469 \text{ for } N=5 \end{cases} \qquad \text{(C-40)}$$

D. TRACK ESTABLISHMENT

An acquired track becomes an established track when a sufficient number of replies, or so-called hits, is accumulated during the 26 interrogations following track acquisition; or a total of 30 interrogations when the initial four used in track acquisition are included. The criteria selected (by MITRE) for MCAS are shown in Table 1 of Section III-E-2.

Track extension statistics are characterized in terms of:

(a) $L(n)$ = minimum number of accumulated replies needed to continue track through the nth interrogation as shown in Table 1 (Section III-E-2).

(b) $H_\ell(n)$ = probability of accumulating ℓ or more replies, with $\ell \geq L(n)$, through the nth interrogation;

(c) $H_\ell(n-1)$ = probability of accumulating ℓ or more replies through n-1 interrogations;

(d) $H_{\ell-1}(n-1)$ = probability of accumulating ℓ-1 or more replies through n-1 interrogations with $\ell-1 \geq L(n-1)$;

(e) $H_{\ell-1}(n-1) - H_\ell(n-1)$ = probability of accumulating exactly ℓ-1 replies through n-1 interrogations;

(f) F = probability of obtaining a reply on any one interrogation;

(g) $[H_{\ell-1}(n-1) - H_\ell(n-1)] \cdot F$ = joint probability of accumulating exactly ℓ-1 replies through n-1 interrogations and one more reply on the nth interrogation; with $\ell-1 \geq L(n-1)$.

Thus, for $\ell-1 \geq L(n-1)$, ℓ or more replies are accumulated with n interrogations, i.e., event (b), through the two mutually exclusive events identified in (c) and (g); hence

$$\begin{aligned} H_\ell(n) &= H_\ell(n-1) + [H_{\ell-1}(n-1) - H_\ell(n-1)]F \\ &= (1-F)H_\ell(n-1) + F\, H_{\ell-1}(n-1) \text{ for } \ell \geq L(n-1) + 1. \end{aligned} \tag{C-41}$$

When $\ell = L(n-1)$, the only way of accumulating ℓ or more replies through n interrogations, without losing track, is through accumulation of ℓ or more replies at the previous, (n-1), interrogation. In other words

$$H_\ell(n) = H_\ell(n-1) \text{ for } \ell = L(n-1) = L(n), \qquad \text{(C-42)}$$

which is invoked only in those case where $L(n-1) = L(n)$ in Table 1 (Section III-E-2). In other words, when no additional replies are required to continue track through a given interrogation, then the probability of maintaining track is the same as that for the preceding interrogation.

The conditional probability of false track continuation, given a false track acquisition, is obtained by using Eq. C-40 for F in Eq. C-41, together with Eq. C-42 and the initial conditions:

$$H_0(1) = 1 \qquad \text{(C-43a)}$$

$$H_1(1) = F. \qquad \text{(C-43b)}$$

Results of computer printouts are shown in Tables C-8a, b, and c. Here, every non-zero entry is the value of $H_\ell(n)$ where n is the row number and ℓ is the column number. The first non-zero entry in any row is the probability that track is continued through that interrogation; the column number for this entry is simply the minimum number of accumulated replies needed to continue track as shown in Table 1. Since a track is declared to be established if it persists for 26 interrogations after acquisition (or 30 interrogations if the first four for acquisition are included), the bottom entry, $H_{13}(26)$, gives the conditional probability of a false track establishment, given an initial false acquisition. Specifically,

$$H_{13}\,(26) = \begin{cases} 0.7211\text{x}10^{-1} & \text{for } N=5 \\ 0.1276\text{x}10^{-2} & \text{for } N=4 \\ 0.6407\text{x}10^{-6} & \text{for } N=3. \end{cases} \qquad \text{(C-44)}$$

TABLE C-8a. PROBABILITY OF ACCUMULATING THE INDICATED NUMBER OF REPLIES AS A FUNCTION OF THE NUMBER OF INTERROGATIONS DURING FALSE TRACK CONTINUATION AFTER ACQUISITION. (PROBABILITY OF A FALSE REPLY PER INTERROGATION = 0.1081 FOR N=3)

Interrogation After Acquisition	Accumulated Replies After Acquisition						
	0	1	2	3	4	5	6
1	0.1000E+01	0.1081E+00	0.	0.	0.	0.	0.
2	0.1000E+01	0.2045E+00	0.1169E-01	0.	0.	0.	0.
3	0.1000E+01	0.2905E+00	0.3253E-01	0.1263E-02	0.	0.	0.
4	0.1000E+01	0.3672E+00	0.6042E-01	0.4643E-02	0.1366E-03	0.	0.
5	0.1000E+01	0.4356E+00	0.9358E-01	0.1067E-01	0.6237E-03	0.1476E-04	0.
6	0.1000E+01	0.4966E+00	0.1306E+00	0.1963E-01	0.1710E-02	0.8059E-04	0.1596E-05
7	0.	0.5510E+00	0.1701E+00	0.3163E-01	0.3648E-02	0.2567E-03	0.1013E-04
8	0.	0.5510E+00	0.2113E+00	0.4660E-01	0.6672E-02	0.6233E-03	0.3679E-04
9	0.	0.5510E+00	0.2480E+00	0.6440E-01	0.1099E-01	0.1277E-02	0.1002E-03
10	0.	0.	0.2808E+00	0.8425E-01	0.1676E-01	0.2327E-02	0.2274E-03
11	0.	0.	0.2808E+00	0.1055E+00	0.2406E-01	0.3887E-02	0.4544E-03
12	0.	0.	0.	0.1244E+00	0.3286E-01	0.6068E-02	0.8255E-03
13	0.	0.	0.	0.1244E+00	0.4276E-01	0.8964E-02	0.1392E-02
14	0.	0.	0.	0.	0.5159E-01	0.1262E-01	0.2211E-02
15	0.	0.	0.	0.	0.	0.1683E-01	0.3336E-02
16	0.	0.	0.	0.	0.	0.	0.4794E-02
17	0.	0.	0.	0.	0.	0.	0.4794E-02
18	0.	0.	0.	0.	0.	0.	0.
19	0.	0.	0.	0.	0.	0.	0.
20	0.	0.	0.	0.	0.	0.	0.
21	0.	0.	0.	0.	0.	0.	0.
22	0.	0.	0.	0.	0.	0.	0.
23	0.	0.	0.	0.	0.	0.	0.
24	0.	0.	0.	0.	0.	0.	0.
25	0.	0.	0.	0.	0.	0.	0.
26	0.	0.	0.	0.	0.	0.	0.

TABLE C-8a. (CONTINUED)

Interrogation After Acquisition	Accumulated Replies After Acquisition 7	8	9	10	11	12	13
1	0.	0.	0.	0.	0.	0.	0.
2	0.	0.	0.	0.	0.	0.	0.
3	0.	0.	0.	0.	0.	0.	0.
4	0.	0.	0.	0.	0.	0.	0.
5	0.	0.	0.	0.	0.	0.	0.
6	0.	0.	0.	0.	0.	0.	0.
7	0.1725E-06	0.	0.	0.	0.	0.	0.
8	0.1249E-05	0.1865E-07	0.	0.	0.	0.	0.
9	0.5092E-05	0.1517E-06	0.2016E-08	0.	0.	0.	0.
10	0.1537E-04	0.6857E-06	0.1820E-07	0.2179E-09	0.	0.	0.
11	0.3829E-04	0.2273E-05	0.9035E-07	0.2161E-08	0.2355E-10	0.	0.
12	0.8327E-04	0.6167E-05	0.3263E-06	0.1169E-07	0.2547E-09	0.2546E-11	0.
13	0.1635E-03	0.1450E-04	0.9577E-06	0.4571E-07	0.1491E-08	0.2980E-10	0.2753E-12
14	0.2963E-03	0.3061E-04	0.2422E-05	0.1443E-06	0.6271E-08	0.1878E-09	0.3467E-11
15	0.5033E-03	0.5933E-04	0.5469E-05	0.3905E-06	0.2119E-07	0.8454E-09	0.2339E-10
16	0.8094E-03	0.1073E-03	0.1129E-04	0.9395E-06	0.6111E-07	0.3045E-08	0.1122E-09
17	0.1240E-02	0.1832E-03	0.2167E-04	0.2059E-05	0.1561E-06	0.9322E-08	0.4293E-09
18	0.1624E-02	0.2975E-03	0.3914E-04	0.4179E-05	0.3617E-06	0.2519E-07	0.1391E-08
19	0.	0.4409E-03	0.6706E-04	0.7958E-05	0.7744E-06	0.6157E-07	0.3963E-08
20	0.	0.4409E-03	0.1075E-03	0.1435E-04	0.1551E-05	0.1386E-06	0.1019E-07
21	0.	0.	0.1435E-03	0.2441E-04	0.2934E-05	0.2913E-06	0.2407E-07
22	0.	0.	0.	0.3729E-04	0.5256E-05	0.5770E-06	0.5296E-07
23	0.	0.	0.	0.	0.8719E-05	0.1083E-05	0.1096E-06
24	0.	0.	0.	0.	0.8719E-05	0.1908E-05	0.2148E-06
25	0.	0.	0.	0.	0.	0.2645E-05	0.3979E-06
26	0.	0.	0.	0.	0.	0.	0.6407E-06

TABLE C-8b. PROBABILITY OF ACCUMULATING THE INDICATED NUMBER OF REPLIES AS A FUNCTION OF THE NUMBER OF INTERROGATIONS DURING FALSE TRACK CONTINUATION AFTER ACQUISITION. (PROBABILITY OF A FALSE REPLY PER INTERROGATION = 0.2180 FOR N=4)

Interrogation After Acquisition	Accumulated Replies After Acquisition 0	1	2	3	4	5	6
1	0.1000E+01	0.2180E+00	0.	0.	0.	0.	0.
2	0.1000E+01	0.3885E+00	0.4752E-01	0.	0.	0.	0.
3	0.1000E+01	0.5218E+00	0.1219E+00	0.1036E-01	0.	0.	0.
4	0.1000E+01	0.6260E+00	0.2090E+00	0.3467E-01	0.2259E-02	0.	0.
5	0.1000E+01	0.7076E+00	0.2999E+00	0.7268E-01	0.9323E-02	0.4924E-03	0.
6	0.1000E+01	0.7713E+00	0.3888E+00	0.1222E+00	0.2313E-01	0.2417E-02	0.1073E-03
7	0.	0.8212E+00	0.4722E+00	0.1803E+00	0.4474E-01	0.6934E-02	0.6109E-03
8	0.	0.8212E+00	0.5483E+00	0.2440E+00	0.7430E-01	0.1517E-01	0.1989E-02
9	0.	0.8212E+00	0.6078E+00	0.3103E+00	0.1113E+00	0.2806E-01	0.4864E-02
10	0.	0.	0.6543E+00	0.3751E+00	0.1547E+00	0.4621E-01	0.9921E-02
11	0.	0.	0.6543E+00	0.4360E+00	0.2027E+00	0.6985E-01	0.1783E-01
12	0.	0.	0.	0.4836E+00	0.2536E+00	0.9882E-01	0.2917E-01
13	0.	0.	0.	0.4836E+00	0.3037E+00	0.1326E+00	0.4435E-01
14	0.	0.	0.	0.	0.3429E+00	0.1699E+00	0.6358E-01
15	0.	0.	0.	0.	0.	0.2076E+00	0.8675E-01
16	0.	0.	0.	0.	0.	0.	0.1131E+00
17	0.	0.	0.	0.	0.	0.	0.1131E+00
18	0.	0.	0.	0.	0.	0.	0.
19	0.	0.	0.	0.	0.	0.	0.
20	0.	0.	0.	0.	0.	0.	0.
21	0.	0.	0.	0.	0.	0.	0.
22	0.	0.	0.	0.	0.	0.	0.
23	0.	0.	0.	0.	0.	0.	0.
24	0.	0.	0.	0.	0.	0.	0.
25	0.	0.	0.	0.	0.	0.	0.
26	0.	0.	0.	0.	0.	0.	0.

TABLE C-8b. (CONTINUED)

Interrogations After Acquisition	Accumulated Replies After Acquisition 7	8	9	10	11	12	13
1	0.	0.	0.	0.	0.	0.	0.
2	0.	0.	0.	0.	0.	0.	0.
3	0.	0.	0.	0.	0.	0.	0.
4	0.	0.	0.	0.	0.	0.	0.
5	0.	0.	0.	0.	0.	0.	0.
6	0.	0.	0.	0.	0.	0.	0.
7	0.2340E-04	0.	0.	0.	0.	0.	0.
8	0.1515E-03	0.5101E-05	0.	0.	0.	0.	0.
9	0.5521E-03	0.3701E-04	0.1112E-05	0.	0.	0.	0.
10	0.1492E-02	0.1493E-03	0.8938E-05	0.2424E-06	0.	0.	0.
11	0.3330E-02	0.4420E-03	0.3954E-04	0.2138E-05	0.5285E-07	0.	0.
12	0.6491E-02	0.1072E-02	0.1273E-03	0.1029E-04	0.5074E-06	0.1152E-07	0.
13	0.1144E-01	0.2253E-02	0.3331E-03	0.3580E-04	0.2640E-05	0.1196E-06	0.2512E-08
14	0.1861E-01	0.4255E-02	0.7517E-03	0.1006E-03	0.9868E-05	0.6692E-06	0.2804E-07
15	0.2842E-01	0.7385E-02	0.1515E-02	0.2425E-03	0.2965E-04	0.2675E-05	0.1678E-06
16	0.4113E-01	0.1197E-01	0.2795E-02	0.5200E-03	0.7606E-04	0.8555E-05	0.7143E-06
17	0.5682E-01	0.1833E-01	0.4795E-02	0.1016E-02	0.1728E-03	0.2327E-04	0.2424E-05
18	0.6909E-01	0.2672E-01	0.7745E-02	0.1840E-02	0.3566E-03	0.5588E-04	0.6968E-05
19	0.	0.3596E-01	0.1188E-01	0.3127E-02	0.6799E-03	0.1214E-03	0.1763E-04
20	0.	0.3596E-01	0.1713E-01	0.5035E-02	0.1213E-02	0.2432E-03	0.4026E-04
21	0.	0.	0.2123E-01	0.7672E-02	0.2047E-02	0.4547E-03	0.8450E-04
22	0.	0.	0.	0.1063E-01	0.3273E-02	0.8017E-03	0.1652E-03
23	0.	0.	0.	0.	0.4876E-02	0.1340E-02	0.3040E-03
24	0.	0.	0.	0.	0.4876E-02	0.2111E-02	0.5299E-03
25	0.	0.	0.	0.	0.	0.2714E-02	0.8747E-03
26	0.	0.	0.	0.	0.	0.	0.1276E-02

TABLE C-8c. PROBABILITY OF ACCUMULATING THE INDICATED NUMBER OF REPLIES AS A FUNCTION OF THE NUMBER OF INTERROGATIONS DURING FALSE TRACK CONTINUATION AFTER ACQUISITION. (PROBABILITY OF A FALSE REPLY PER INTERROGATION = 0.3469 FOR N=5)

Interrogation After Acquisition	Accumulated Replies After Acquisition						
	0	1	2	3	4	5	6
1	0.1000E+01	0.3469E+00	0.	0.	0.	0.	0.
2	0.1000E+01	0.5735E+00	0.1203E+00	0.	0.	0.	0.
3	0.1000E+01	0.7214E+00	0.2775E+00	0.4175E-01	0.	0.	0.
4	0.1000E+01	0.8181E+00	0.4315E+00	0.1235E+00	0.1448E-01	0.	0.
5	0.1000E+01	0.8812E+00	0.5656E+00	0.2304E+00	0.5231E-01	0.5024E-02	0.
6	0.1000E+01	0.9224E+00	0.6751E+00	0.3467E+00	0.1141E+00	0.2143E-01	0.1743E-02
7	0.	0.9493E+00	0.7609E+00	0.4606E+00	0.1948E+00	0.5357E-01	0.8572E-02
8	0.	0.9493E+00	0.8262E+00	0.5648E+00	0.2870E+00	0.1026E+00	0.2418E-01
9	0.	0.9493E+00	0.8689E+00	0.6555E+00	0.3833E+00	0.1665E+00	0.5137E-01
10	0.	0.	0.8968E+00	0.7295E+00	0.4777E+00	0.2417E+00	0.9132E-01
11	0.	0.	0.8968E+00	0.7876E+00	0.5651E+00	0.3236E+00	0.1435E+00
12	0.	0.	0.	0.8255E+00	0.6423E+00	0.4074E+00	0.2060E+00
13	0.	0.	0.	0.8255E+00	0.7058E+00	0.4889E+00	0.2758E+00
14	0.	0.	0.	0.	0.7473E+00	0.5641E+00	0.3497E+00
15	0.	0.	0.	0.	0.	0.6277E+00	0.4241E+00
16	0.	0.	0.	0.	0.	0.	0.4947E+00
17	0.	0.	0.	0.	0.	0.	0.4947E+00
18	0.	0.	0.	0.	0.	0.	0.
19	0.	0.	0.	0.	0.	0.	0.
20	0.	0.	0.	0.	0.	0.	0.
21	0.	0.	0.	0.	0.	0.	0.
22	0.	0.	0.	0.	0.	0.	0.
23	0.	0.	0.	0.	0.	0.	0.
24	0.	0.	0.	0.	0.	0.	0.
25	0.	0.	0.	0.	0.	0.	0.
26	0.	0.	0.	0.	0.	0.	0.

TABLE C-8c. (CONTINUED)

Interrogations After Acquisition	Accumulated Replies After Acquisition 7	8	9	10	11	12	13
1	0.	0.	0.	0.	0.	0.	0.
2	0.	0.	0.	0.	0.	0.	0.
3	0.	0.	0.	0.	0.	0.	0.
4	0.	0.	0.	0.	0.	0.	0.
5	0.	0.	0.	0.	0.	0.	0.
6	0.	0.	0.	0.	0.	0.	0.
7	0.6045E-03	0.	0.	0.	0.	0.	0.
8	0.3368E-02	0.2097E-03	0.	0.	0.	0.	0.
9	0.1059E-01	0.1305E-02	0.7275E-04	0.	0.	0.	0.
10	0.2474E-01	0.4526E-02	0.5004E-03	0.2524E-04	0.	0.	0.
11	0.4783E-01	0.1154E-01	0.1897E-02	0.1901E-03	0.8755E-05	0.	0.
12	0.8102E-01	0.2413E-01	0.5241E-02	0.7821E-03	0.7165E-04	0.3037E-05	0.
13	0.1244E+00	0.4386E-01	0.1179E-01	0.2329E-02	0.3181E-03	0.2684E-04	0.1054E-05
14	0.1769E+00	0.7179E-01	0.2292E-01	0.5612E-02	0.1016E-02	0.1279E-03	0.9999E-05
15	0.2369E+00	0.1083E+00	0.3987E-01	0.1162E-01	0.2610E-02	0.4358E-03	0.5089E-04
16	0.3018E+00	0.1529E+00	0.6360E-01	0.2142E-01	0.5734E-02	0.1190E-02	0.1844E-03
17	0.3687E+00	0.2045E+00	0.9457E-01	0.3605E-01	0.1117E-01	0.2766E-02	0.5333E-03
18	0.4124E+00	0.2615E+00	0.1327E+00	0.5635E-01	0.1980E-01	0.5683E-02	0.1308E-02
19	0.	0.3139E+00	0.1774E+00	0.8284E-01	0.3248E-01	0.1058E-01	0.2826E-02
20	0.	0.3139E+00	0.2247E+00	0.1156E+00	0.4995E-01	0.1818E-01	0.5516E-02
21	0.	0.	0.2557E+00	0.1535E+00	0.7274E-01	0.2920E-01	0.9909E-02
22	0.	0.	0.	0.1889E+00	0.1008E+00	0.4430E-01	0.1660E-01
23	0.	0.	0.	0.	0.1313E+00	0.6389E-01	0.2621E-01
24	0.	0.	0.	0.	0.1313E+00	0.8728E-01	0.3928E-01
25	0.	0.	0.	0.	0.	0.1026E+00	0.5593E-01
26	0.	0.	0.	0.	0.	0.	0.7211E-01

E. ALARM GENERATION

The conditional probability that a falsely established track generates a false alarm is the probability that the track meets the threat criteria in range, range rate, and altitude. At any given time, the range vs. range-rate criterion (i.e., alarm when the projected time to collision is approximately 30 sec) will be met by all tracks whose initial (at start of acquisition) range and range rate fell within the shaded region in Fig. C-2. Any track initiated at the lower boundary will have 30 sec for track establishment (4 sec for acquisition plus 26 sec for subsequent extensions before becoming established) and will therefore alarm, if the altitude criterion (to be discussed below) is also met, at 0 sec "time-to-go". Similarly, any track initiated at the upper boundary will alarm* at 30 sec "time-to-go". Tracks initiated below the shaded region would be receding from the interrogator before they were established while the contribution of tracks initiated above the shaded region (i.e., those which have persisted for more than 26 extensions) are neglected. Since the initial ranges and range rates are uniformly distributed (see discussion between Eq. C-3 and Eq. C-4) within the 20 nmi x 1650 ft/sec rectangle, the probability that an established track was initiated in the shaded region (in Fig. C-2) is

$$\frac{\frac{1}{2} \times 8.14 \text{ nmi} \times 1650 \text{ ft/sec}}{20 \text{ nmi} \times 1650 \text{ ft/sec}} = 0.2035. \qquad (45)$$

The distribution of the apparent altitude of a surviving false track is determined from the following considerations:

1. The track initiation procedure, which involves the "anding" of the garble data within the false bracket quadruplet, will tend to "thin out" the number of

*This neglects the contributions from minimum range alarms.

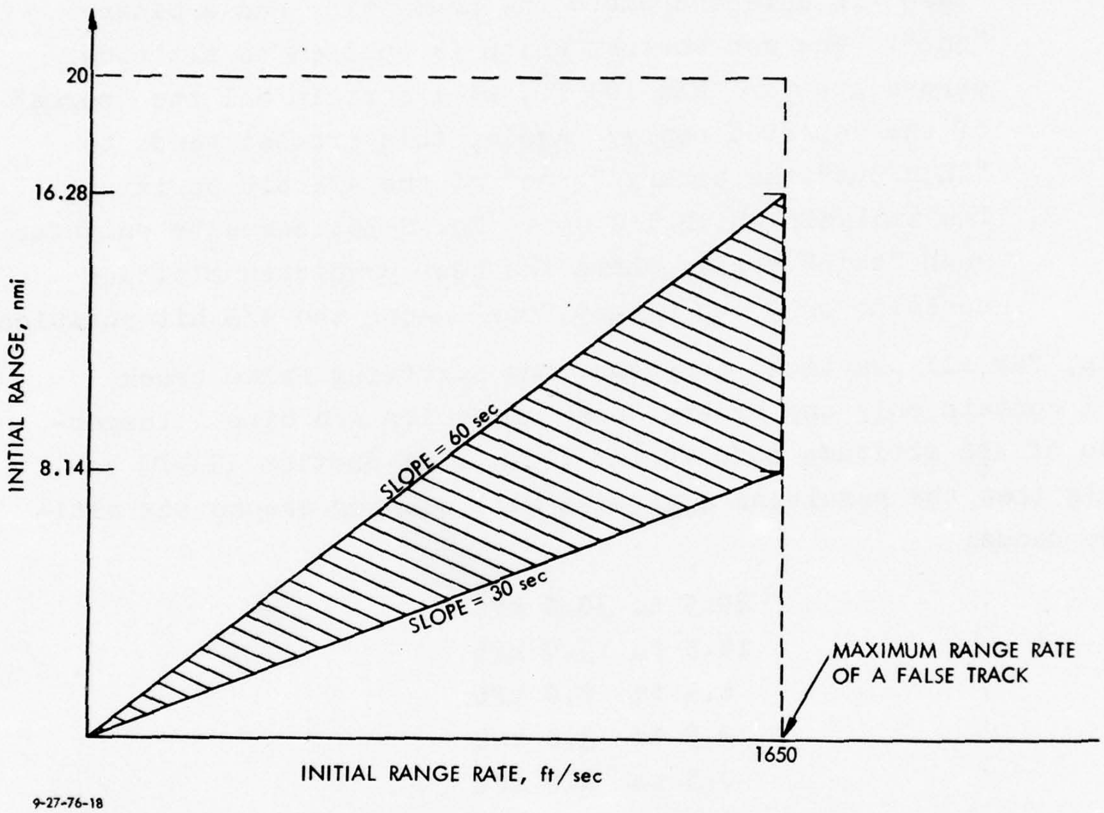


FIGURE C-2. Initial Range and Range Rate of False Track Initiations

binary "ones". In other words, the "anded" data produces a binary "one" only when it has been detected within each of the four brackets.

2. During subsequent track extensions, the tracker logic introduces a corrected track altitude every time a "zero" is detected where the prediction had a binary "one". The correction, which is applied to altitude errors greater than 100 ft, will contain all the "zeros" of the detected reply. Again, this process tends to "thin out" the binary "ones" of the A/B bit positions.
3. The analysis which led up to Eq. C-36, accounts only for such "thin" tracks where the best predicted altitude contains only one binary "one" among the A/B bit positions.

Thus, for all practical purposes, the surviving false track will contain only one binary "one" among its A/B bits. Inspection of the altitude code format (Fig. 2 of Section III-D) reveals that the resultant altitudes will cluster around six altitude bands:

29.5 to 30.0 kft
14.5 to 15.0 kft
6.5 to 7.0 kft
2.5 to 3.0 kft
0.5 to 1.0 kft
-0.5 to 0.0 kft

with equal probability, i.e., 1/6. Thus, any BCAS interrogator within ±600 ft (for altitudes below 10,000 ft) or within ±800 ft (for altitudes above 10,000 ft) of any one of the six bands would, in accordance with ANTC-117, receive dive or climb commands; and climbing/diving rate restrictions for altitudes within ±3400 ft of the six bands. These criteria, however, presume cooperative and complementary warnings/commands, which is not the case in a BCAS-to-ATCRBS encounter. In this instance, the altitude threat bands would have to be expanded to

account for the unpredictable altitude changes by the ATCRBS aircraft. In any case, given a BCAS interrogator within several hundred feet of one of the six bands, listed above, the probability of a false alarm will be 1/6 if the false track also meets the range and range rate criteria. The probability for the latter is given by Eq. C-45 so that

$$\text{Conditional probability of false alarm, given that a false track was established} = 0.2035 \times 1/6 = 0.0339 \quad (C-46)$$

The overall (unconditional) false alarm rate is the product of:

1. The rate of false track acquisitions as computed in Table C-1 (bottom row).
2. The conditional probability of false track establishment, given a false track acquisition; and this is given by Eq. C-44.
3. The conditional probability of false alarm, Eq. C-46, given that a false track was established.

The results are shown in Table C-9 as a function of

N = number of overlapping replies
= number of responding a/c per 1.64 nmi range increment

so that

$$\text{number of responding aircraft within 20 nmi} = \frac{20}{1.64} N. \quad (C-46a)$$

Introduction of "whisper-shout" (cf. Section III-G), would subdivide the aircraft population into four equal nonoverlapping groups, provided that the projected improvements can, in fact, be achieved. Thus, if N is the number of overlapping replies in any one group then the results of Table C-9 would apply to each group. Consequently,

TABLE C-9. COMPUTATION OF THE FALSE ALARM RATE

Statistic	Number of Overlapping Replies, N N=3	N=4	N=5	Method of Computation
Expected number of false track acquisitions per second	0.0561	1.84	21.0	Table C-1, bottom row
Conditional probability of false track establishment, given false acquisition	0.6407×10^{-6}	1.276×10^{-3}	7.211×10^{-2}	Eq. C-44 (obtained from Tables 8a, b, c
Conditional probability of false alarm, given that a false track was established	0.0339	0.0339	0.0339	Eq. C-46
Expected false alarm rate, alarms per hour, experienced by one interrogator	4.39×10^{-6}	0.287	185	Product of above, times 3600 sec/hr

(1) the total number of aircraft in the four groups would be four times the number given by Eq. C-46a, and

(2) the overall false alarm rate for the four groups would be four times that given in Table C-9 (bottom row).

These results are summarized in Table C-10. Thus, even if the false alarm rate of 1.1 per hour were acceptable, the corresponding traffic volume, i.e., 196 aircraft within 20 nmi, is still short of the 412 aircraft projected for the Los Angeles terminal area (FAA projection for 1982).

TABLE C-10. FALSE ALARM RATES AS A FUNCTION OF AIRCRAFT POPULATION*

No. of overlapping replies	Without "whisper-shout"		With "whisper-shout"	
	No. of aircraft within 20 nmi	False alarms per hr	No. of aircraft within 20 nmi	False alarms per hr
3	37	4.4×10^{-6}	148	1.8×10^{-5}
4	49	0.29	196	1.1
5	61	185	244	740

*See analysis below Eq. C-46a.

F. TRACK CAPACITY

The conditional average duration, λ, of a false track, given a false track acquisition, is obtained as follows:

(1) Let f(n) denote the conditional probability that a false track was maintained for n or more interrogations, given a false track initiation; f(n) is simply the first non-zero entry in the nth row of Tables C-8a, b, and c. In terms of the formulation used in Section D, $f(n) = H_{L(n)}(n)$.

(2) In terms of (1), the probability that a false track is maintained for n + 1 or more interrogations is f(n+1).

(3) The probability that a false track is maintained for exactly n interrogations is the difference between (1) and (2), i.e., $f(n) - f(n+1)$.

Consequently, the average track life, λ, is

$$\lambda = \sum_{n=1}^{\infty} n[f(n) - f(n+1)]$$

$$= \sum_{n=1}^{\infty} nf(n) - \sum_{n=1}^{\infty} nf(n+1) . \qquad (47)$$

The summation index, n, in the second sum is replaced by k-1 so that

$$\lambda = \sum_{n=1}^{\infty} nf(n) - \sum_{k=2}^{\infty} (k-1)f(k)$$

$$= \sum_{n=1}^{\infty} nf(n) - \sum_{k=2}^{\infty} kf(k) + \sum_{k=2}^{\infty} f(k) \qquad (C\text{-}48)$$

$$= f(1) + \sum_{n=2}^{\infty} nf(n) - \sum_{k=2}^{\infty} kf(k) + \sum_{k=2}^{\infty} f(k).$$

Since the first and second summations cancel, and since f(1) can be combined with the last summation by extending its summation index down to k = 1, we obtain

$$\lambda = \sum_{k=1}^{\infty} f(k) \qquad (C\text{-}49)$$

Using the f(k) from Tables C-8a, b, and c (the first non-zero entry in any given row), and neglecting contributions beyond k = 26, Eq. C-49 yields

$$\lambda = \begin{cases} 8.54 \text{ sec for } N=3 \\ 11.7 \text{ sec for } N=4 \\ 16.1 \text{ sec for } N=5 \end{cases} \qquad (C\text{-}50)$$

The number of false tracks in the track file is estimated from the simplified "cookie-cutter" model shown in Fig. C-3. Here, the length of a horizontal line is the sum of:

(1) The conditional average duration, λ, of a false track, given that a false track was acquired.

(2) The track acquisition time of 3 sec needed for four interrogations.

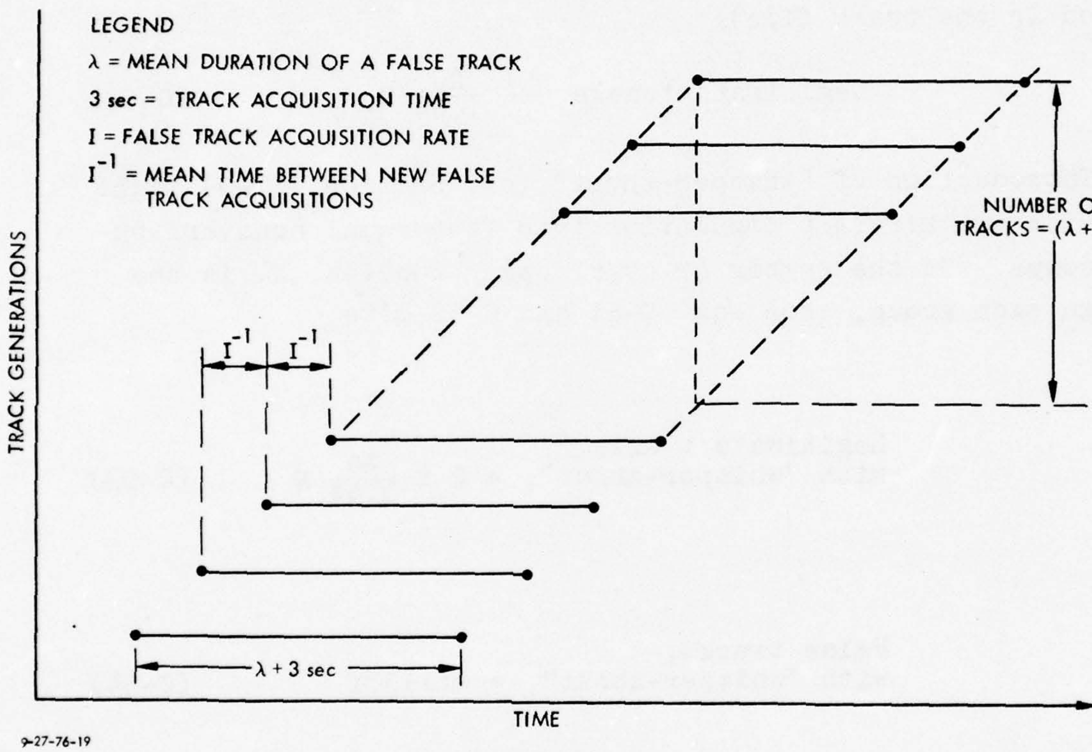


FIGURE C-3. Computation of the Expected Number of False Tracks in the Track File

If I = false track initiation rate, as computed in the bottom row of Table C-1, then $(\lambda+3)I$ is the number of new false tracks acquired during the life time, $\lambda+3$, of any one false track. Thus, after the initial $\lambda+3$ sec, the average number of false tracks is

$$\text{False tracks} = (\lambda+3)I \tag{C-51}$$

while the number of legitimate tracks is just one-half of Eq. C-46a (the other half is assumed to be receding and is not entered in the track file):

$$\text{Legitimate tracks} = \frac{1}{2}\,\frac{20}{1.64}\,N \tag{C-52}$$

Introduction of "whisper-shout" (cf. Section III-G) might subdivide the aircraft population into four equal nonoverlapping groups. If the number of overlapping replies, N, is the same in each group, then Eqs. C-51 and C-52 give

$$\begin{array}{l}\text{Legitimate tracks,}\\ \text{with "whisper-shout",}\end{array} = 2 \times \frac{20}{1.64}\,N \tag{C-53}$$

and

$$\begin{array}{l}\text{False tracks,}\\ \text{with "whisper-shout",}\end{array} = 4(\lambda+3)I \tag{C-54}$$

where λ is given by Eq. C-50, and I, the false track initiation rate, is given in the bottom row of Table C-1.

The track capacity calculations, together with the false alarm rates computed in Table C-10, are summarized in Table C-11.

TABLE C-11. SUMMARY OF FALSE ALARM RATES AND TRACK LOAD ESTIMATES (WITH "WHISPER-SHOUT")

Performance Characteristics	Number of overlapping replies, N: N=3	N=4	N=5	Method of Computation
Number of aircraft (within 20 nmi) producing the indicated reply overlap	148	196	244	Table C-10
False alarms per hour, per interrogator, produced by the indicated reply overlap	1.8×10^{-5}	1.1	740	Table C-10
Number of legitimate tracks	74	98	122	One-half* of the number of aircraft within 20 nmi
Legitimate tracks <u>plus</u> a lower bound on false tracks**	77	206	1726	Sum of above and Eq. C-54
Track capacity of present MCAS prototype	≈ 200			Informal estimate by MITRE

*The other half of the aircraft are assumed to be receding from the interrogator and are therefore not entered in the track file.

**This is an expected value calculation and does not include peak loading effects nor the multiple track branches generated by each track, legitimate or false.

G. EFFECTS OF A MODIFIED AIRCRAFT POPULATION PROFILE

When there are no responding aircraft within 1.6 nmi of a BCAS interrogator, then the preceding results are modified as follows. First, Eqs. C-3, C-4, and C-6 are each multiplied by

$$\frac{20 \text{ nmi} - 1.6 \text{ nmi}}{20 \text{ nmi}} = 0.92 \qquad \text{(C-55)}$$

which means that rows (A) and (D) of Table C-1 as well as the top row of Table C-11 are, each, multiplied by 0.92.

Secondly, some of the false tracks which survived up to 1.6 nmi of the interrogator will be terminated within one or two interrogations depending on the accumulated false replies prior to this point. Thus, a lower bound on the number of false tracks which could produce a false alarm are those which were initiated in the shaded region in Fig. C-4 (which replaces Fig. C-2). For example: a false track initiated at 3.2 nmi with an apparent range rate of 324 ft/sec will have 30 sec for track establishment. At the end of this time it will be 1.6 nmi from the interrogator, if the track survives, with an apparent time-to-collision of

$$\frac{1.6 \text{ nmi} \times 6080 \text{ ft/nmi}}{324 \text{ ft/sec}} = 30 \text{ sec},$$

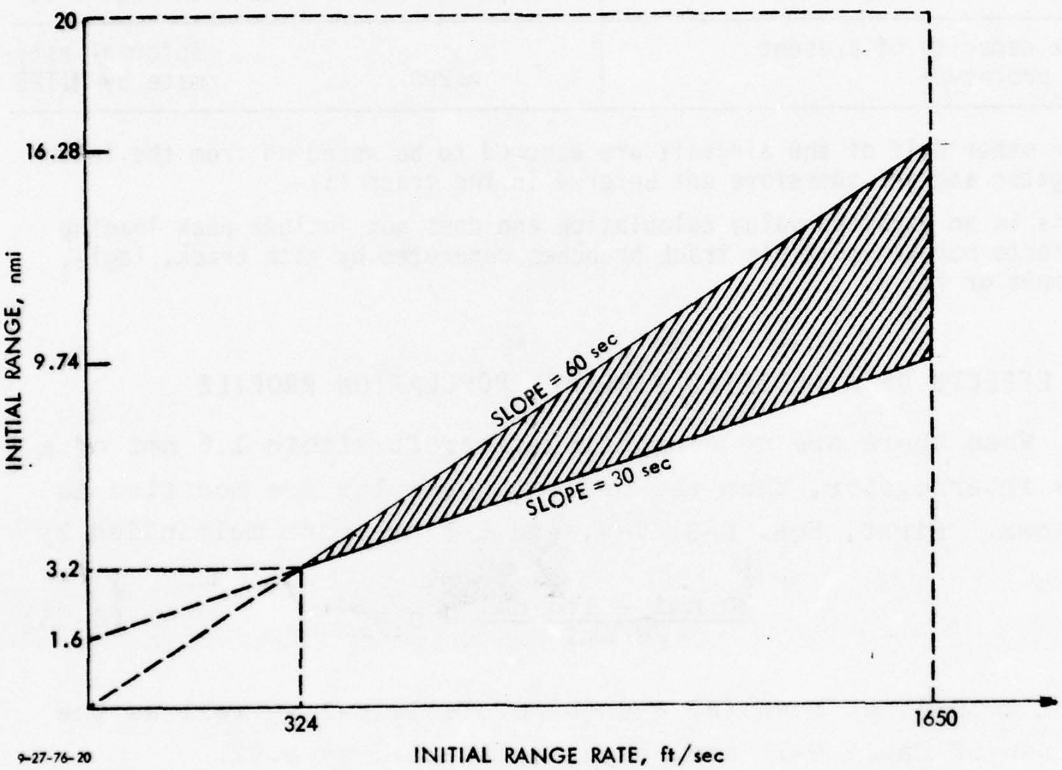


FIGURE C-4. Initial Range and Range Rate of False Track Initiations for Modified Aircraft Population (Not to Scale).

and this is within the alarm boundary. Similarly, a false track initiated at 9.74 nmi and an apparent range rate of 1650 ft/sec will, after 30 sec of track continuation, reach within 1.6 nmi of the interrogator with a projected time-to-collision of

$$\frac{1.6 \text{ nmi} \times 6080 \text{ ft/nmi}}{1650 \text{ ft/sec}} = 6 \text{ sec}$$

In general, any false track initiated in the shaded region in Fig. C-4, and surviving for 30 sec, (needed for track establishment) will, at a 1.6 nmi range, have a projected time-to-collision of 6 sec to 30 sec, depending on where the track was started within the shaded region.

Since track starts are uniformly distributed between 1.6 nmi to 20 nmi and 0 to 1650 ft/sec (see discussion relating to Fig. C-2), the probability of a start within the shaded region in Fig. C-4 is

$$\frac{\frac{1}{2}\,(1650 \text{ ft/sec} - 324 \text{ ft/sec}) \times 6.54 \text{ nmi}}{1650 \text{ ft/sec} \times (20 \text{ nmi} - 1.6 \text{ nmi})} = 0.143 \qquad \text{(C-56)}$$

Thus the ratio of Eq. C-56 to Eq. C-45, multiplied by Eq. C-55:

$$\frac{0.143}{0.2035} \times 0.92 = 0.65, \qquad \text{(C-57)}$$

gives the factor by which the false alarm rates, shown in Table C-11, would be reduced if all aircraft within 1.6 nmi of the interrogator were removed. However, if these removed aircraft were redistributed uniformly between 1.64 nmi and 20 nmi, in order to preserve the total aircraft population, then the reply overlap would have to be increased by a factor which is the reciprocal of Eq. C-55. Thus, if original reply overlap were four, then the new overlap becomes (4/0.92) = 4.35. Because of the steep increase in false alarm rates with reply overlap

(see Table C-11), an exponential interpolation between $N = 4$ and $N = 5$ is used to obtain the false alarm rate at $N = 4.35$. Thus, the new false alarm rate, at $N = 4.35$, is the antilog of (refer to Table C-11 for false alarm rates at $N = 4$ and $N = 5$),

$$\log 1.1 + \frac{\log 740 - \log 1.1}{5 - 4} (4.35-4),$$

multiplied by Eq. C-57. The resultant false alarm rate becomes 7.0 alarms per hour compared to the original 1.1 (see Table C-11 for $N = 4$). Thus, in spite of the fact that the alarm rate decreases by 35 percent (see Eq. C-57) when the close-in aircraft, (within 1.6 nmi) are removed, the overall false alarm rate will increase almost seven times when these same aircraft are redistributed uniformly between 1.6 and 20 nmi.

APPENDIX D

MULTIPATH CONSIDERATIONS

The stylized multipath geometry shown in Fig. D-1 is used to derive the plots in Fig. D-2. These are terminated at 8 nmi, the assumptions being that:

1. At the maximum instrumented BCAS range, 20 nmi, the direct signal will be 6 db above the MDL (=minimum detectable level); and
2. The multipath level is 14 db below the direct signal level (the ANTC-117 specifies a 10 db multipath level) so that beyond 8 nmi, multipath is below the MDL.

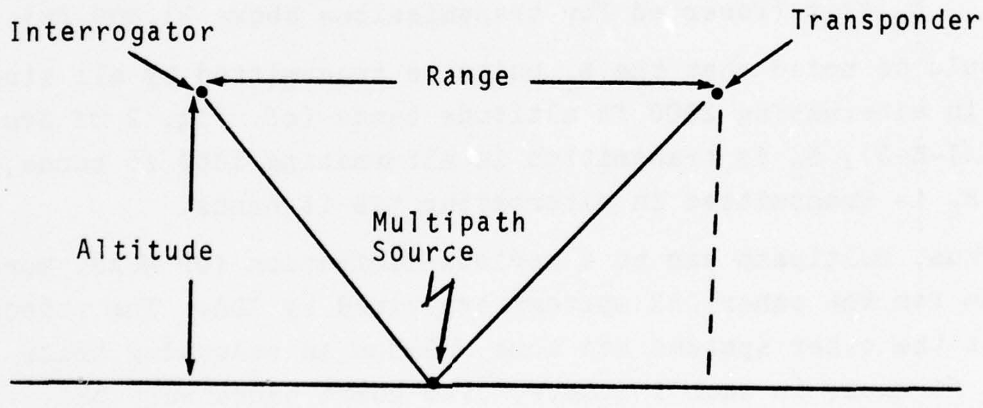


FIGURE D-1. Multipath Geometry

The significant points to be made about Fig. D-2 are as follows:

A. Above the 0.45 μsec curve (corresponding to the pulse width), the multipath component becomes an additional source of garble. In effect, aircraft within 8 nmi of the interrogator will contribute twice as much garble as the aircraft beyond 8 nmi.

B. Between 1.225 μsec and 1.675 μsec, the multipath propagated pulse will arrive in the slot allocated to the subsequent pulse of the altitude code (see Fig. D-2 of main text). Thus, for example, a B_2 pulse arriving by multipath will appear in the D_2 slot (reserved for aircraft above 63,000 ft) and is therefore decoded as an aircraft above 63,000 ft. Similarly, a B_4 pulse will appear in the D_4 slot assigned to transmissions above 31,000 ft.

C. Between 4.125 μsec and 4.575 μsec (see Fig. D-2) the multipath pulse is delayed by three slot intervals (3x1.45 μsec = 4.25 μsec). Thus, for example, a B_1 pulse arriving by multipath appears in the D_2 slot while a B_2 pulse arriving by multipath appears in the D_4 slot (reserved for transmissions above 31,000 ft).

It should be noted that the B_1 pulse is transmitted by all aircraft in alternating 2000 ft altitude bands (cf. Fig. 2 of Section III-E-2), B_2 is transmitted in alternating 1000 ft bands, while B_4 is transmitted in alternating 500 ft bands.

Thus, multipath can be a serious limitation for MCAS, more so than for the other CAS systems evaluated by IDA. The reason is that the other systems had some freedom in selecting their signal formats; in each instance, time guard bands were provided to alleviate the multipath interference problem. In contrast, MCAS is limited to the ATCRBS format which was not intended for the multipath geometries encountered in air-to-air CAS applications.

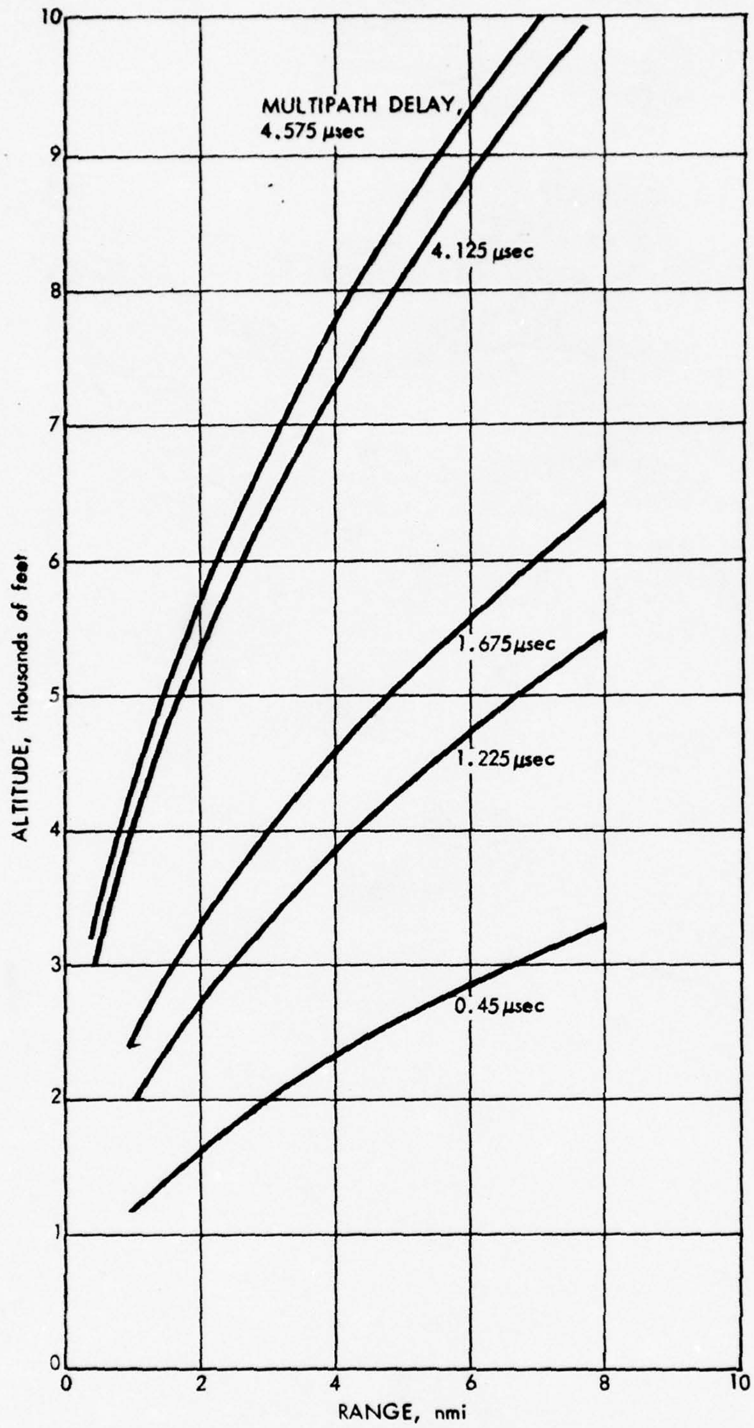


FIGURE D-2. Plots of Altitude versus Range Producing the Indicated Multipath Delays. (Multipath source is assumed to be at the midpoint).

APPENDIX E

ESTIMATE OF AVERAGE NUMBER PULSES TRANSMITTED IN RESPONSE TO BCAS INTERROGATIONS

An MCAS transponder will respond to an interrogation by transmitting two framing pulses (F_1 and F_2) separated in time by 20.3 μsec and a number of altitude code pulses which are spaced at 1.45 μsec intervals within the time separation between the two framing pulses. Thirteen pulse positions are possible between the two framing pulses; however, only twelve of these are used for altitude encoding. The seventh pulse position is designated as the "X" pulse and is not used by the BCAS. An ATCRBS transponder may also transmit, upon command from the ground, a "SPI" pulse after transmission of the second framing pulse (i.e., F_2). The "SPI" pulse is not used by the BCAS system.

The ATCRBS altitude transmission code is shown in Fig. E-1. The code consists of two Gray codes: one with a 500-foot increment, and the other with a 100-foot increment. The 100-foot increment code is transmitted in pulse positions C4, C2 and C1; and the 500-foot increment code uses pulse positions B4, B2, B1, A4, A2, A1, D4, and D2. (The D1 position is not used). The position of these bits in the reply pulse transmission sequence is shown at the bottom of Fig. E-1.

Eight code values are possible with the three "C" bits; however, only five codes are used. (The code values of 0, 5 and 7 are not used.) We will assume that aircraft are randomly distributed over the 500-ft unambiguous altitude interval of the "C" bits so that, on the average, 1.4 "C" bit pulses are

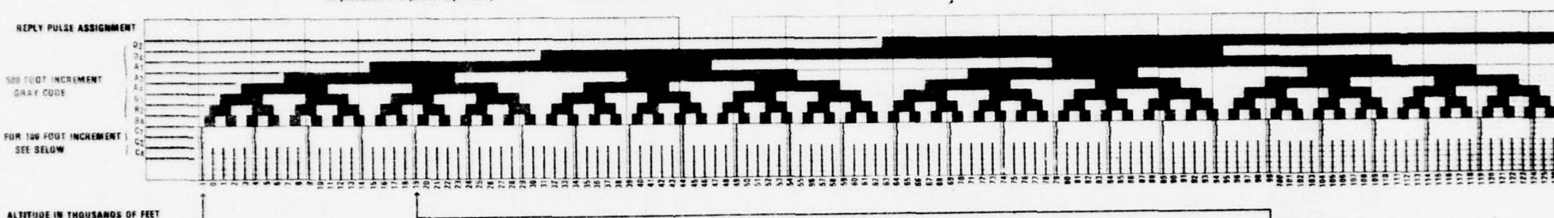
UNIT DISTANCE REFLECTED BINARY CODE FOR 8 BITS

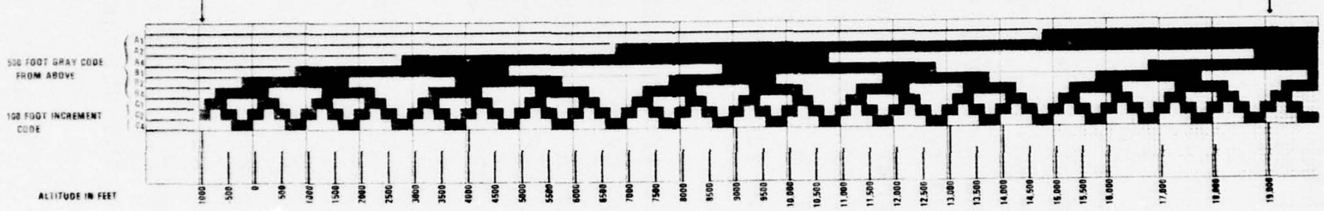
A4, B1, B2, B4, PULSES* (columns); D2, D4, A1, A2 PULSES* (rows)

	0000	0001	0011	0010	0110	0111	0101	0100	1100	1101	1111	1110	1010	1011	1001	1000
0000	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0001	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16
0011	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47
0010	63	62	61	60	59	58	57	56	55	54	53	52	51	50	49	48
0110	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79
0111	95	94	93	92	91	90	89	88	87	86	85	84	83	82	81	80
0101	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111
0100	127	126	125	124	123	122	121	120	119	118	117	116	115	114	113	112
1100	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143
1101	159	158	157	156	155	154	153	152	151	150	149	148	147	146	145	144
1111	160	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175
1110	191	190	189	188	187	186	185	184	183	182	181	180	179	178	177	176
1010	192	193	194	195	196	197	198	199	200	201	202	203	204	205	206	207
1011	223	222	221	220	219	218	217	216	215	214	213	212	211	210	209	208
1001	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239
1000	255	254	253	252	251	250	249	248	247	246	245	244	243	242	241	240

256 INCREMENTS (500 FOOT EACH) GIVING ALTITUDE FROM -1000 FEET TO 127,000 FEET

*0 or 1 in a pulse position indicates the absence or presence of a pulse respectively.





100 FOOT INCREMENT TABLE		
100 FOOT PULSES	SUM OF DIGITS OF GRAY CODE	
C_1 C_2 C_4	ODD	EVEN
0 0 1	7	8
0 1 1	6	9
0 1 0	5	0
1 1 0	4	1
1 0 0	3	2

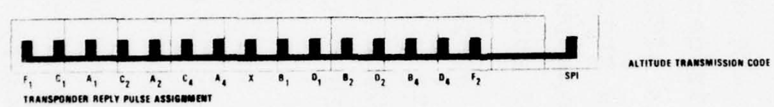


FIGURE E-1. Altitude Transmission Code

transmitted. The unambiguous altitude interval of the B4 bit is 2000 ft and that of the B2 bit is 4000 ft. We will assume that the aircraft are randomly distributed over these bit intervals so that the B2 and B4 bits can be expected to contribute another pulse to the average pulse transmission rate. So far, the average pulse transmission rate consists of 2 framing pulses: 1.4 "C" bit pulses and 1 B2/B4 pulse, giving a total of 4.4 pulses per interrogation. This does not include the B1, A4, A2, A1, D4, or D1 bits. The unambiguous altitude intervals of these bits are large so that their expected occurrence will be sensitive to the altitude distribution of the aircraft.

The altitude distribution of the FAA static traffic model for the 1982 LA Basin is presented in Table E-1. Also shown in the table is the altitude at which the higher-order altitude bits are transmitted. For example, the B1 bit is transmitted if the altitude of the aircraft is between 1000 ft and 5000 ft; between 9000 ft and 12,000 ft; between 17,000 ft and 21,000 ft; or between 26,000 ft and 29,000 ft. In the table, the altitudes of the aircraft are quantized in 500-ft increments below 10,000 ft and in 1000-ft increments above 10,000 ft. The changes in the altitude bits coincide with the quantization of the aircraft's altitudes and can result in a high estimate in the number of bits transmitted. For example, the table lists 40 aircraft at 1000-ft altitude. If all of these aircraft are exactly at 1000-ft altitude, they will all transmit a B1 altitude bit. However, if half of the aircraft are between 1000 ft and 1250 ft and the other half are between 750 ft and 1000 ft, only 20, rather than 40, B1 bits will be transmitted by this group of aircraft. To take this effect into account, the number

of pulses transmitted at a code transition altitude is assumed to be an average of the number of high order bit pulses transmitted at the quantized altitude above and below the transmission altitude. Also, to simplify the calculation, all aircraft above 30,000 ft are assumed to transmit an average of 2.5 high-order altitude bits.

There are 797 aircraft listed in Table E-1, and an average of 1275.5 high-order altitude bits would be transmitted. The per aircraft average is therefore 1.6 pulses. Adding this to the low order bits and the framing pulse gives an average of 6.0 pulses per aircraft per transmission.

TABLE E-1. ALTITUDE DISTRIBUTION OF AIRCRAFT IN THE 1982 L.A. BASIN STATIC TRAFFIC MODEL

Altitude (kft)	Transmitted Altitude Pulses					No. High order Pulse transmits per aircraft	No. Aircraft	Total Pulses Transmitted
	B1	A4	A2	A1	D4			
0.5						0	1	0
1.0						.5	40	20
1.5						1	64	64
2.0						1	38	38
2.5						1	54	54
3.0						1.5	38	57
3.5						2	67	134
4.0						2	3	6
4.5						2	59	118
5.0						1.5	5	7.5
5.5						1	58	58
6.0						1	9	9
6.5						1	58	58
7.0						1.5	7	11.5
7.5						2	58	116
8.0						2	5	10
8.5						2	26	52
9.0						2.5	1	2.5
9.5						3	35	105
10						3	8	24
11						2.5	16	40
12						2	21	42
13						1.5	16	24
14						1	22	22
15						1.5	15	22.5
16						2	20	40
17						2.5	10	25
18						3	5	15
19						3.5	4	14
20						4	2	8
21						3.5	0	0
22						3	1	3
23						2.5	3	7.5
24						2	3	6
25						2.5	7	17.5
26						3	4	12
27						2.5	4	10
28						2	3	6
29						1.5	1	1.5
30-60						2.5	6	15
TOTAL							797	1275.5

APPENDIX F

EFFICACY OF WHISPER-SHOUT AND RESUPPRESSION IN REDUCING EFFECTS OF GARBLE

A. GENERAL

Synchronous reply interference or synchronous garble is recognized as a severe problem in the MITRE BCAS. MITRE has devoted considerable effort in devising a tracker mechanization that will, it is claimed, permit tracking an individual intruder in a group of as many as eight replying aircraft within ± 1.644 nmi ($\pm 20.3\mu s$) of the given intruder. It should be noted that eight aircraft over ± 1.644 nmi corresponds to four over a range interval of 1.644. All future references to aircraft density will mean the latter, i.e. number per 1.644 nmi range increment.

MITRE has not estimated the reliability of decoding altitude messages in such a condition, nor has MITRE supplied any statistically significant experimental evidence that reliable altitude data can be expected in such a condition. In high density traffic predicted for the Los Angeles (LAX) Basin in 1982, it is expected that the condition of such garbling levels will be exceeded over considerable areas. Consequently, MITRE has proposed using the whisper-shout technique to partition the population in any range interval. Lincoln Laboratories has also proposed a different technique, called resuppression, to further partition the group resulting from whisper-shout. MITRE has proposed using both techniques simultaneously. It is expected that, in an operational BCAS, whisper-shout and resuppression would each provide a four-to-one partition so that the overall

effect would be a sixteen-to-one partition. Presumably, if the tracker can accommodate up to four aircraft within a range increment of 1.644 nmi then whisper-shout and resuppression would permit as many as 64 aircraft within a range increment of 1.644 nmi.

B. RESUPPRESSION

Lincoln Laboratories' resuppression scheme is similar to the whisper-shout in that a sequence of interleaved suppression interrogation pulse pairs and mode-C interrogation pulse pairs are transmitted by MCAS. The separation between the suppression pair and the mode-C pair are varied to selectively interrogate the different groups of transponders. The operation depends on the tolerances that exist in the recovery of the transponders to a suppression pair. The specification on recovery is 25-45 μs. Thus, if the first suppression pair and interrogation pair were separated by 30 μs, only those transponders whose recovery were less than 30 μs would respond to the following mode-C interrogation pair. For the second group, two suppression pairs separated by 30 μs are transmitted. These two suppression pairs effectively suppress the first group which is suppressed by each of the suppressions. The two suppression pairs are followed by a mode-C interrogation pair after a delay of about 5 μs. Only those transponders which recover after the first pulse of the second suppression pair and before the first pulse of the mode-C interrogation pair reply in this group.

Subsequent groups are similarly stimulated to respond using two suppression interrogation pairs followed by mode-C interrogation pairs. Thus, the population is divided into groups by virtue of the recovery times following a suppression interrogation pair. It has been observed that some transponders do not operate as expected inasmuch as the second suppression pair may suppress the transponder for longer times than expected. Also, data does not exist on the stability of the suppression time

as the variation with incident power level (or range for a constant interrogation power level) which may reduce the effectiveness of this technique. Lincoln Laboratories has estimated that the groups could be separated by as little as 2 μs, but presented little data to support this contention.

C. MEASURED SUPPRESSION DATA AND INTERROGATION SEQUENCES

Measured suppression data on 448 general aviation ATCRBS transponders given by Colby & Crocker cannot be approximated well by the more common distributions (linear and normal). Figures F-1, F-2 and F-3 show the same data plotted on linear, normal and log-normal paper. The linear fit seems to be as appropriate as the others. The population can be broken up into four equal parts for suppression times of approximately 29, 33 and 38 μsec. It will be assumed that the suppression time is independent of power level, but, clearly, if suppression time is used to separate transponders into groups, data on power sensitivity and suppression stability must be collected before an MCAS that uses it is designed.

The tails of the distribution cause some difficulties. On the low side, one transponder had a suppression time of only 8μs and 5% had suppression times of less than 25μs. However, on the high side the tail is very long. There was one transponder that did not recover until 1.4ms and 5% exceeded 50μs. To assure that no transponder replies more than once the separation between the second suppression interrogation pair and the mode-C interrogation pair should not exceed 8μs (the shortest suppression recovery of the population). However, using 8μs* implies that 170 sequences of two suppression interrogations and mode-C interrogations must be used for the tail. If as few as 1% of the transponders were permitted to respond more than

*Because there are several modes to which transponders will respond, e.g., Mode 3/A with 8μs spacing between P_1 and P_3 and Mode C with 21μs spacing, it will be necessary to avoid some specific spacings between suppressions since a transponder may recover for the P_2 of the first suppression and interpret the next P_1 or P_2 of a suppression as the P_3 of a valid interrogation. Such an effect is important as a source of synchronous garble but has been ignored here.

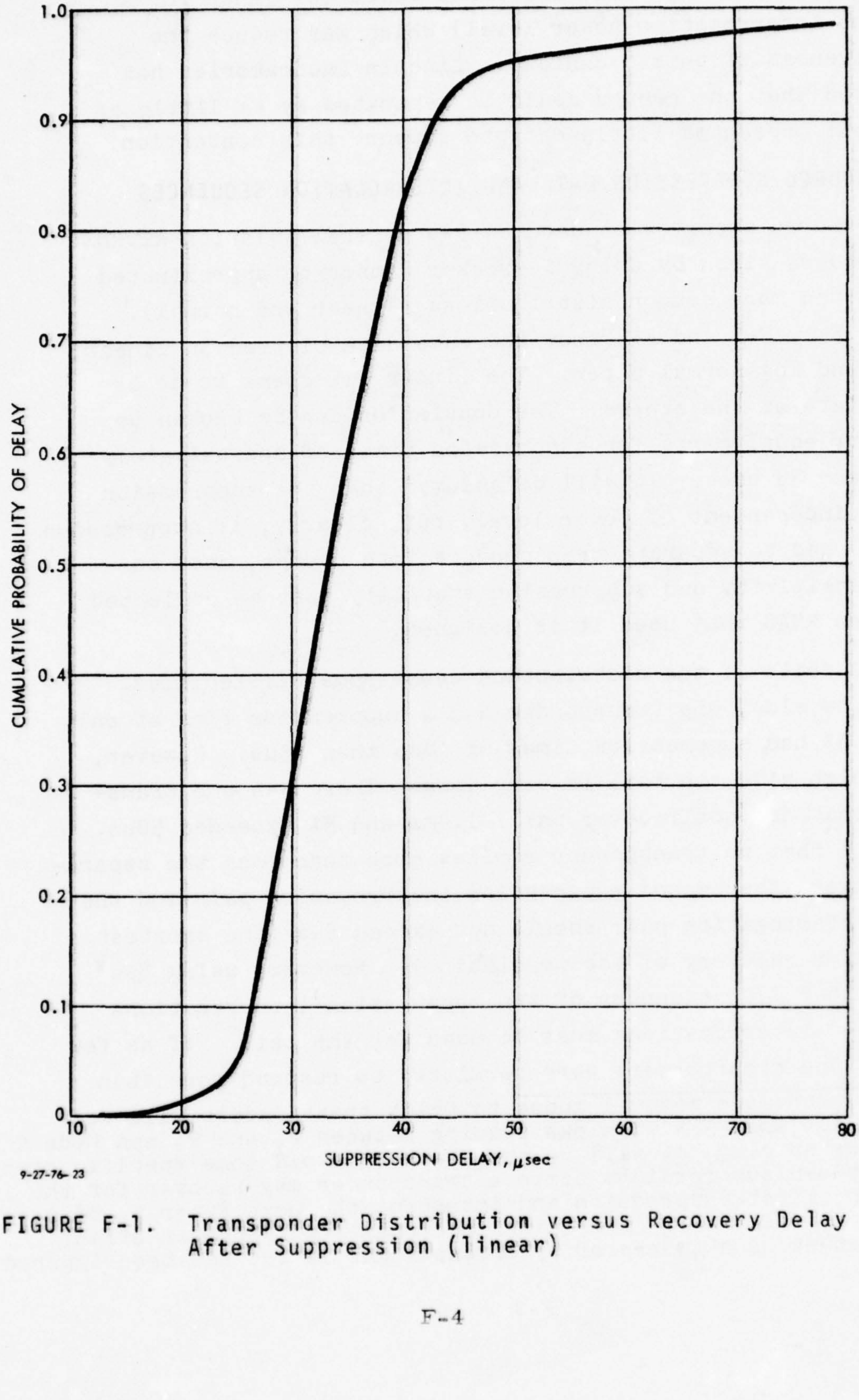


FIGURE F-1. Transponder Distribution versus Recovery Delay After Suppression (linear)

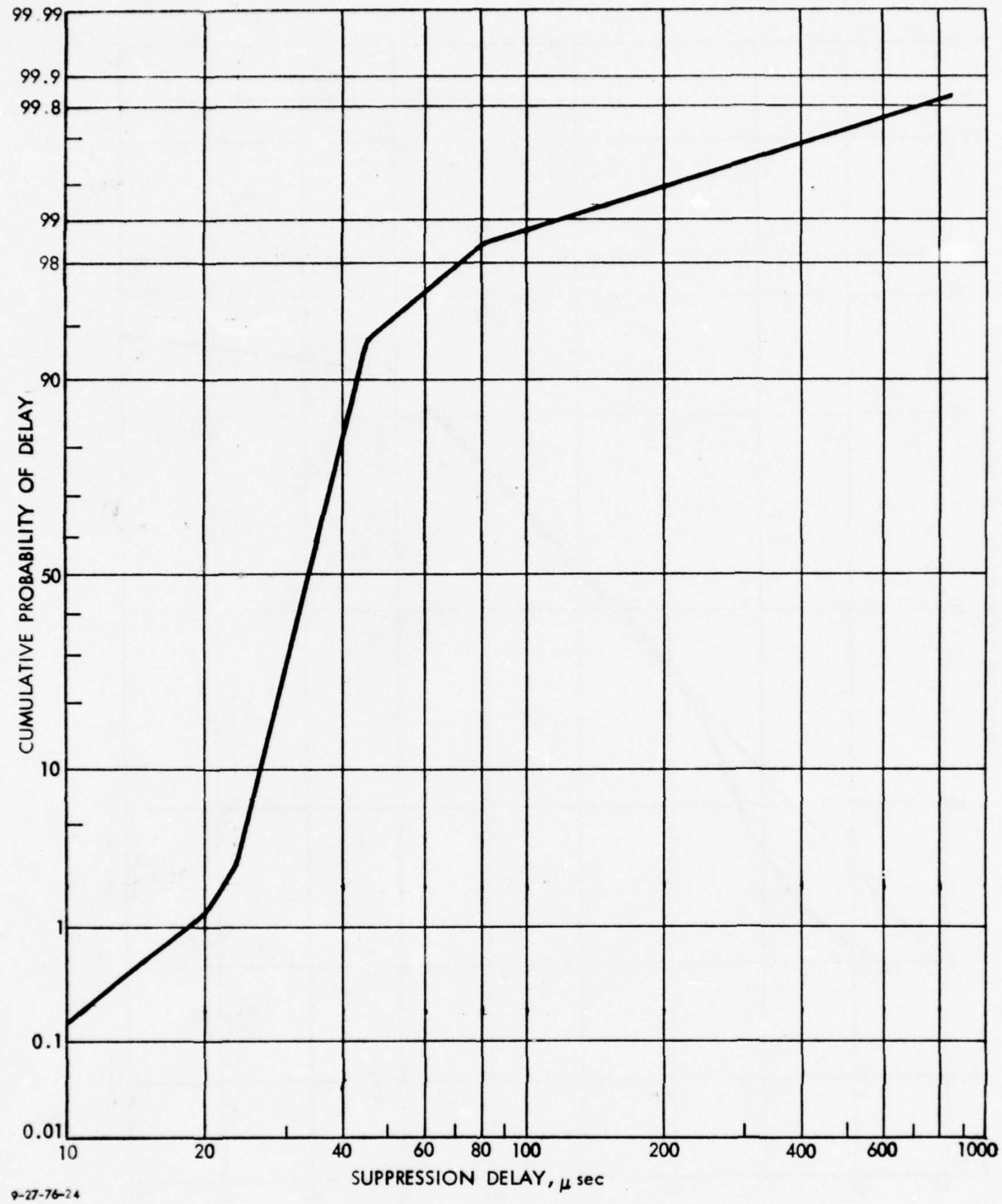


FIGURE F-2. Transponder Distribution versus Recovery Delay After Suppression (log-normal)

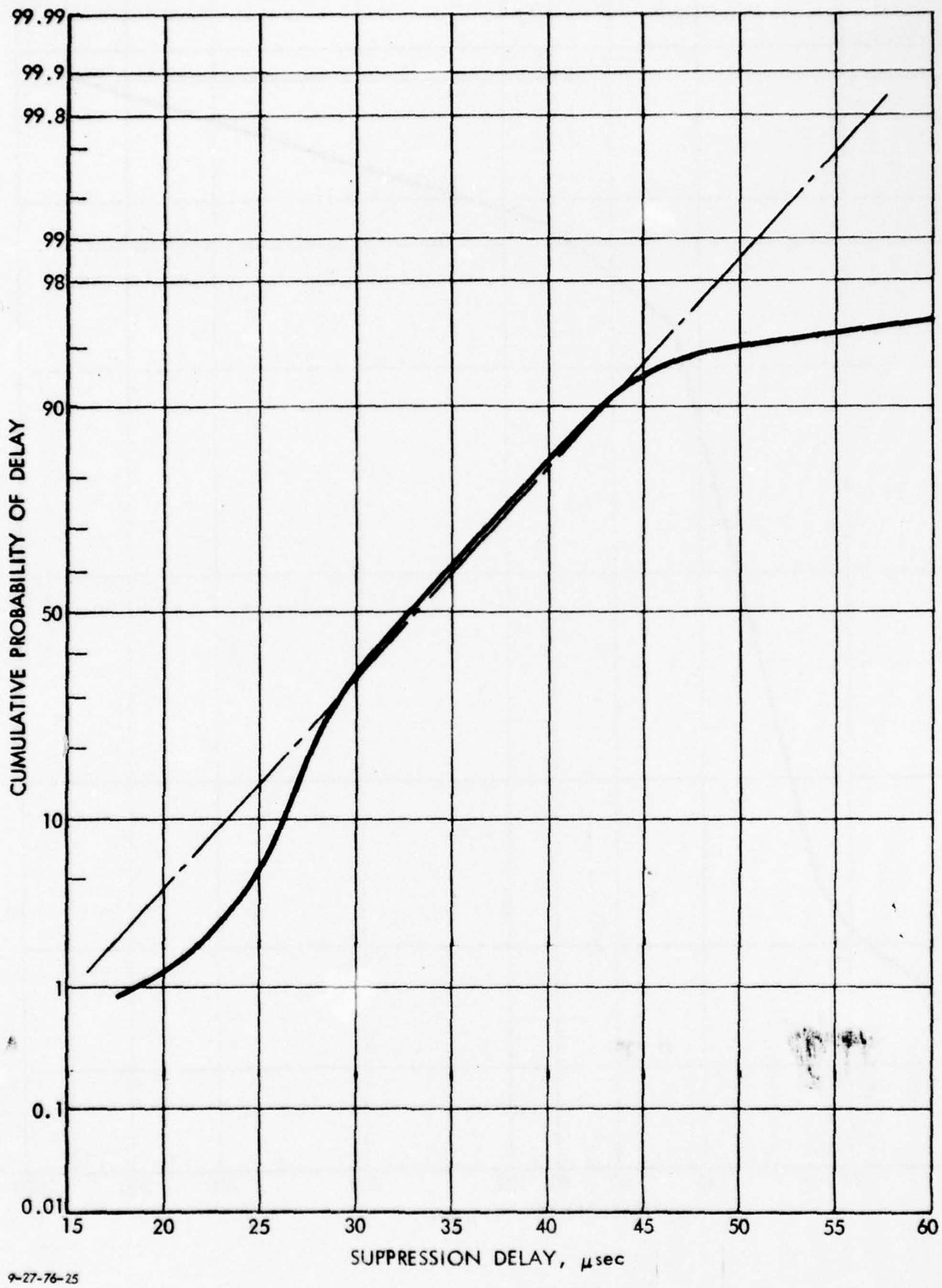


FIGURE F-3. Transponder Distribution versus Recovery Delay After Suppression (normal)

once, then this delay could be increased to about 20μs but still 68 sequences must be used for the tail. If one were satisfied with ignoring, i.e. not interrogating the 0.9% of the tail beyond about 110μs, then the delay of 20μs would require only four sequences for the tail.

In summary, the long tail for suppression recovery delays greater than the specified maximum value of 45μs requires increasing the number of sequences from a nominal value of 4 to 7 and a total of 13 suppression interrogation pulse pairs and 7 mode-C pulse pairs would be required. With such a sequence approximately 0.9% of the transponders would not reply and about 1% would reply more than once. Table F-1 summarizes the corresponding populations. Note that in the interest of reducing sequences further by extending the delay between the second suppression and the mode-C interrogation there is little to be gained because the fraction of the population that replies more than once increases so rapidly from 20μs (1%) to 29μs (25%). The only real alternative is to not interrogate more transponders as shown in Table F-2.

The only solution to the long tails appears to be a program to mandate that transponders meet the specification and then police the transponders.

D. TRAFFIC DENSITY AND TRANSPONDER SENSITIVITY DISTRIBUTIONS

1. Traffic Model

The traffic density model that IDA has used in previous airborne CAS studies is based on the FAA's prediction of traffic in the Los Angeles (LAX) Basin for 1982. Figure F-4 shows the cumulative distribution of traffic from LAX airport for aircraft below 10,000 ft and for all aircraft. A functional representation of the model is also included in Figure F-4. Figure F-5 shows the density of aircraft in an incremental range of 1.644 nmi, corresponding to the ATCRBS reply duration of 20.3μs. The peak density of about 41 aircraft occurs in the

TABLE F-1. SUMMARY OF RESUPPRESSION POPULATIONS

	Fraction of Population				
Group	Incremental	Cumulative	Population Delay Range, μs	Delay for Second Suppression*	Delay for Mode C*
1	0.25	0.25	0-29	0	29
2	0.25	0.50	29-33	29	33
3	0.25	0.75	33-38	33	38
4	0.215	0.964	38-58	38	58
5	0.020	0.984	58-78	58	78
6	0.005	0.987	78-98	78	98
7	0.004	0.991	98-108	98	108
Not interrogated	0.009		108-1398		

*Delay measured from first suppression in a sequence. (Note that for Group 1 there is only one suppression).

TABLE F-2. SUMMARY OF MISSED TRANSPONDERS VERSUS NUMBER OF GROUPS. (20 μs delay for tails)

Number of Groups	Fraction Not Interrogated	Delay Range (μs) of Those Not Interrogated
4	0.036	58-1398
5	0.016	78-1398
6	0.013	98-1398
7	0.009	108-1398

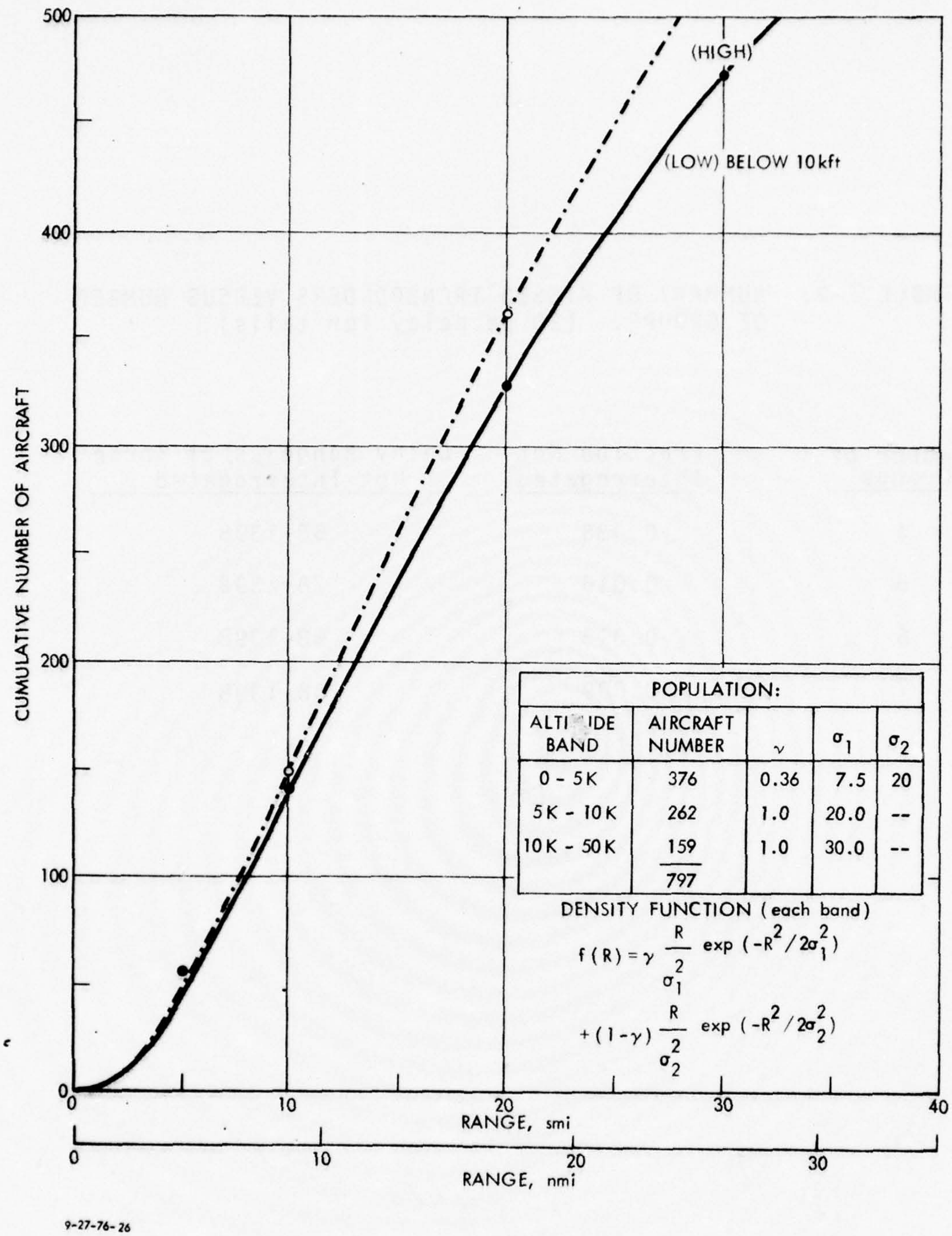


FIGURE F-4. Cumulative Distribution of Aircraft versus Range (FAA - 1982 LAX Projection)

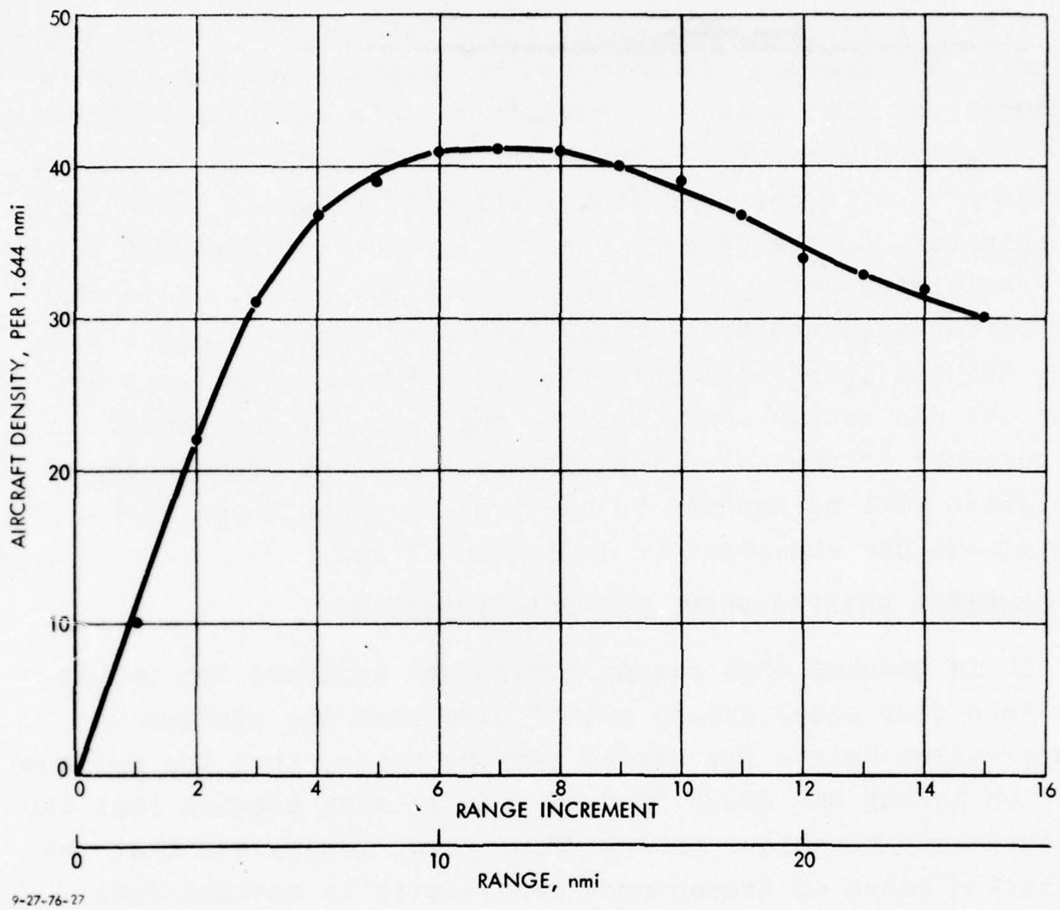


FIGURE F-5. Density of Aircraft versus Range (FAA - 1982 LAX Projection)

vicinity of 10 nmi; the average density is about 35 aircraft between 0 and 20 nmi.

2. Transponder Sensitivity

The specified sensitivity of the ATCRBS transponders is nominally 71 dbm with limits between 69 and 77 dbm. Colby and Cocker have given measured sensitivity data on 505 transponders, including 463 general aviation and 42 military and air carrier transponders. The measured sensitivity data in db approximately are normally distributed with a mean of -71.3 dbm and standard deviation of 6.16 db. The distribution is slightly skewed with a longer tail on the poorer sensitivity side. Figure F-6 shows the sensitivity data plotted on normal graph paper. More than half of the measured transponders deviated from the specification; 38% had poorer sensitivities than the -69 dbm limit and about 14% had better sensitivities than the -77 dbm limit. For purposes of the following calculations, the transponder population will be assumed to be normally distributed with a mean of -71 dbm and standard deviation of 6db.

E. COMBINED WHISPER-SHOUT AND RESUPPRESSION

It is assumed that resuppression can separate the population into four equal groups or, if more than the minimum resuppression delays are needed for the tails, that the maximum fraction in any one group is 0.25. It is also assumed that the whisper-shout technique employs four power levels and that the population based on transponder sensitivity is divided into subgroups at a range of 20 nmi as follows:

Power, dbw	Fractional Subpopulation	Cumulative Population
+6	0.02	0.02
+12	0.14	0.16
+18	0.34	0.50
+30	0.48	0.98
Not responding	0.02	1.00

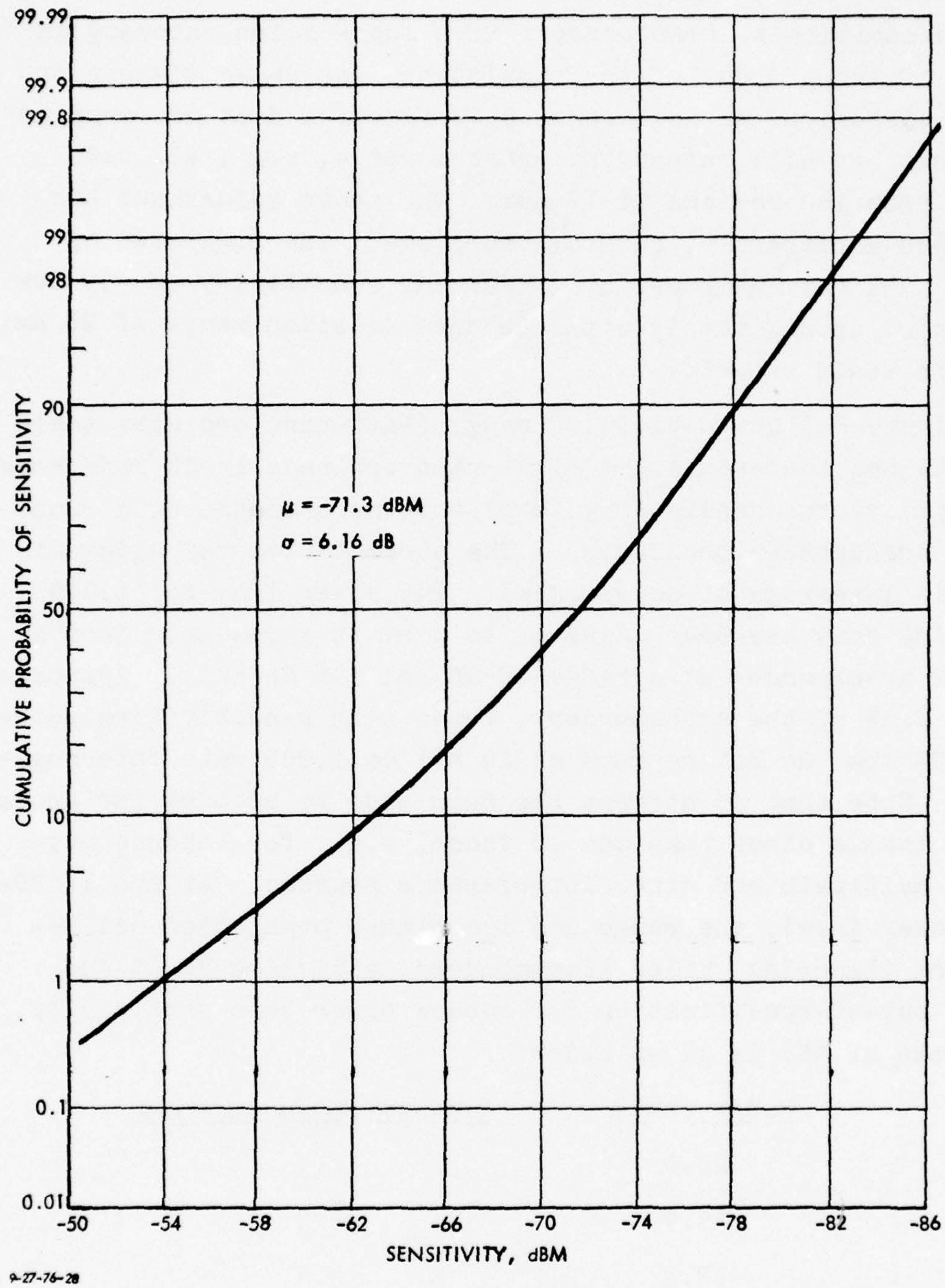


FIGURE F-6. Sensitivity Distribution of 500 Transponders (GA, AC and MIL) After Colby and Crocker

In addition to transponders with suppression recovery in excess of 108μs (.009 of the population, see above discussion on resuppression) an additional 0.02 fraction does not respond at 20 nmi but will respond at shorter range, e.g., all but 0.0008 fraction respond at 10 nmi. The above adjustment has been made arbitrarily, but follows closely one suggested by MITRE. The maximum power of 30 dbw and sensitivity of -59 dbw correspond approximately with the communication range of 20 nmi that one would expect.

Figure F-7 shows plots of range (left-hand ordinate scale in 1.644 nmi increments and right-hand ordinate in db referenced to 1 nmi) versus sensitivity (dbm) and the corresponding cumulative transponder population. The parameter on the diagonal lines is interrogator power (dbw). The power line for 1,000 watts (30 dbw) has been adjusted to provide a response from a -59 dbw transponder at a range of 20 nmi (26 db nmi). Approximately 2.3% of the transponders, those with sensitivities poorer than -59 dbm, do not respond at 20 nmi to 1,000 watt interrogations. Note that no attempt has been made to account for propagation losses other than due to range, e.g., for antenna patterns, multipath and other interference sources. At the 1,000-watt power level, the range and fractional population not responding (ignoring failed transponders, estimated at 6% and others out-of-specification for causes other than sensitivity, estimated at 4%) is given below:

Range (nmi)	Percent of Population
20.0	2.3
15.9	1.0
13.3	0.5
11.2	0.25
9.4	0.1

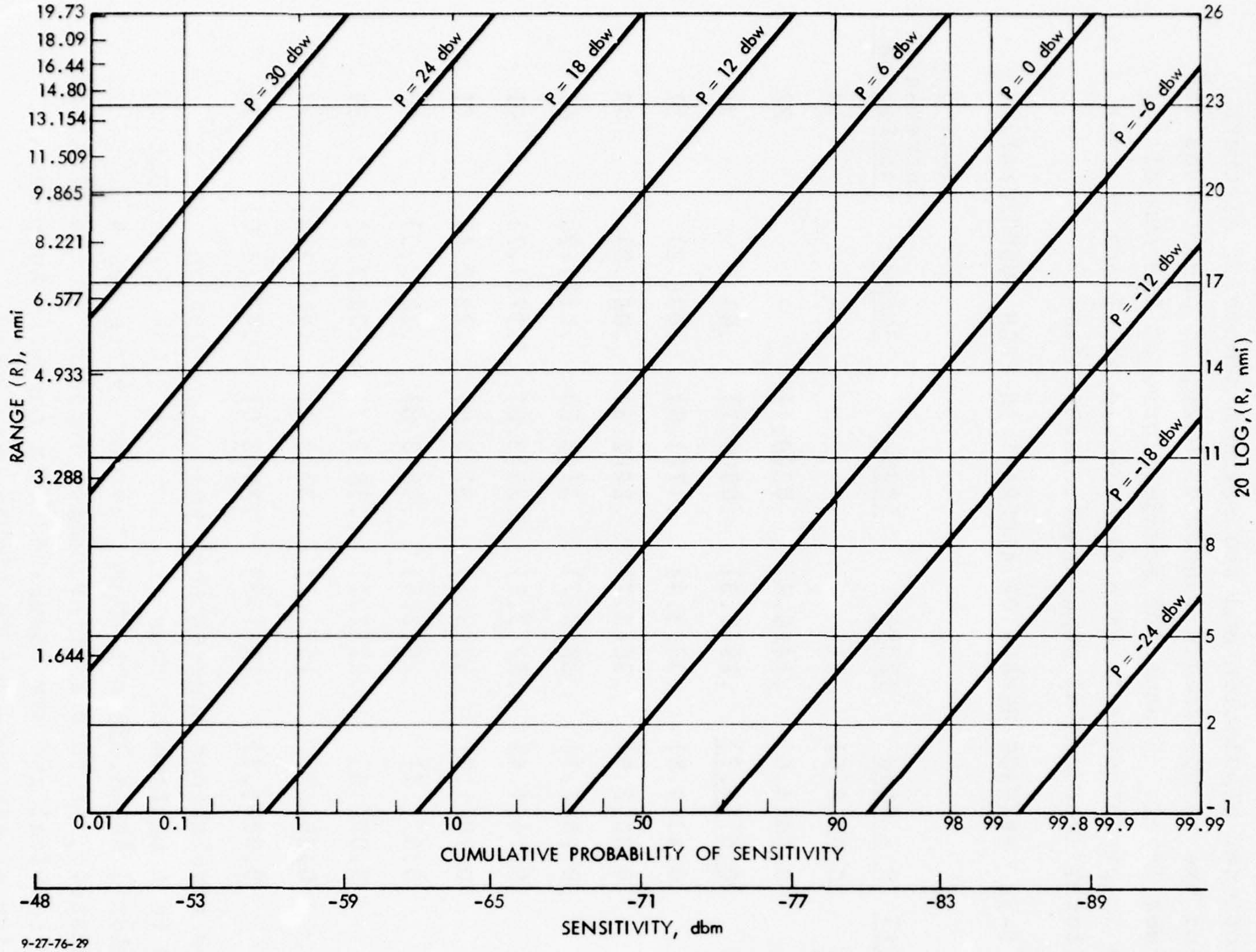


FIGURE F-7. Range/Sensitivity Distributions (Parameter: Interrogator Power)

The following table taken from Figures F-6 and F-7 gives the fractional population and the maximum number of aircraft, including the resuppression factor of 0.25, in parentheses for each power level. Except for roundoff errors, the sum of the numbers in parentheses in each line should equal one quarter of the total incremental population, last column.

TABLE F-3. AVERAGE NUMBER OF AIRCRAFT IN EACH SUBPOPULATION

Group (i)	P - 6dbw	12dbw	18dbw	30dbw	Total A/C Incremental Population
1	0.99(2.5)	.01	-	-	10
2	0.84(4.6)	.14(0.8)	.02(0.1)	-	22
3	0.61(4.7)	.29(2.3)	.09(0.7)	.01	31
4	0.43(4.0)	.37(3.4)	.17(1.6)	.03(.2)	37
5	0.30(2.9)	.38(3.7)	.24(2.3)	.08(.8)	39
6	0.20(2.1)	.36(3.7)	.31(3.2)	.13(1.3)	41
7	0.14(1.4)	.34(3.5)	.35(3.6)	.17(1.7)	41
8	0.10(1.0)	.30(3.1)	.37(3.8)	.23(2.4)	41
9	0.07(.7)	.25(2.5)	.38(3.8)	.30(3.0)	40
10	0.05(.5)	.21(2.1)	.38(3.7)	.36(3.5)	39
11	0.35(.3)	.18(1.7)	.37(3.4)	.415(3.8)	37
12	0.025(.2)	.15(1.3)	.35(3.0)	.475(4.0)	34

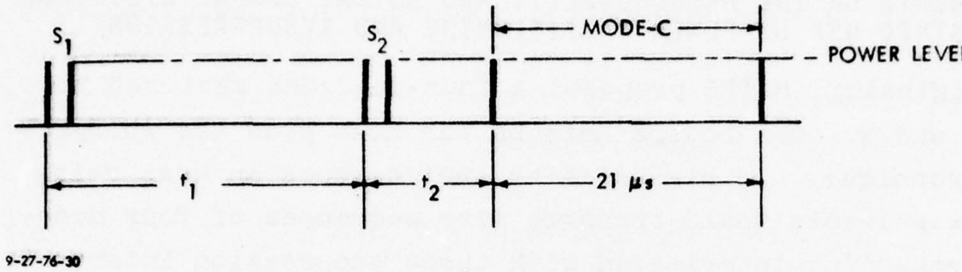
The tabulations suggest that the subpopulation can be limited to a maximum of four aircraft per 1.644 nmi except in the region 1.644-4.933 nmi where the number reaches 4.7. Unfortunately, at this shorter range, decoding of the altitude is very critical and the tabulated data suggests that the estimation of altitude may be highly unreliable.

F. COMMENTS ON THE ORTHOGONALITY AND MUTUAL COMPATIBILITY OF COMBINED USE OF POWER PARTITIONING AND RESUPPRESSION

Originally, MITRE proposed a four-quadrant sectored top antenna and an onmi bottom antenna for MCAS plus the whisper-shout technique. In high-density environments an MCAS using four power levels would transmit five sequences of four mode-C interrogations interleaved with three suppression interrogations, for a total of 20 mode-C interrogations and 15 suppression interrogations. With the sectored antenna it was assumed that garble in a given range cell could be suppressed by at least a factor of two and that the four power levels would provide an additional factor of 2-4 for an effective reduction of garble in a range cell of 4-8 times. If such reductions were deemed inadequate for high-density traffic it was suggested to increase the number of power levels.

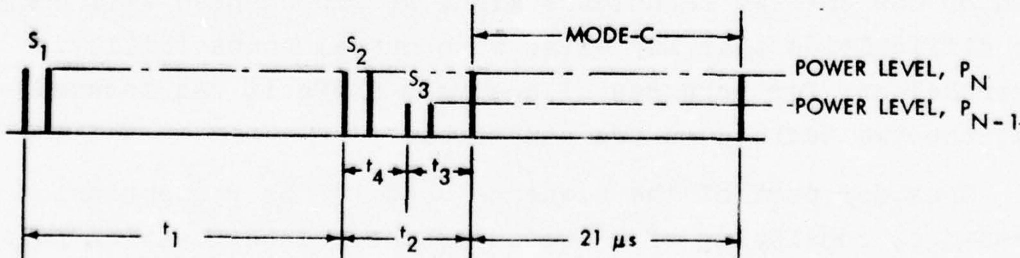
Because of the general dissatisfaction with the garble-reduction potential of the sectored antenna and the perceived difficulty in installation of such an antenna, it has been abandoned and the Lincoln Laboratory resuppression scheme has been substituted. With the latter technique it is assumed that the aircraft population can be divided so that a resuppression group at a range cell would have less than 25% of the population and combined with the four power levels the overall garble-reduction factor would be between 8-16. MITRE has not explained exactly how the two techniques would be used, but some discussion of how the two techniques might be implemented will suggest the difficulties that may exist with mutual compatibility. Nevertheless, for purposes of analysis above it was assumed that the two techniques are compatible.

Consider part of the sequence used in the resuppression technique, consisting of a two suppression interrogation pulse pairs separated by a delay t_1 and followed by a mode-C interrogation pulse pair delayed by t_2 as follows:



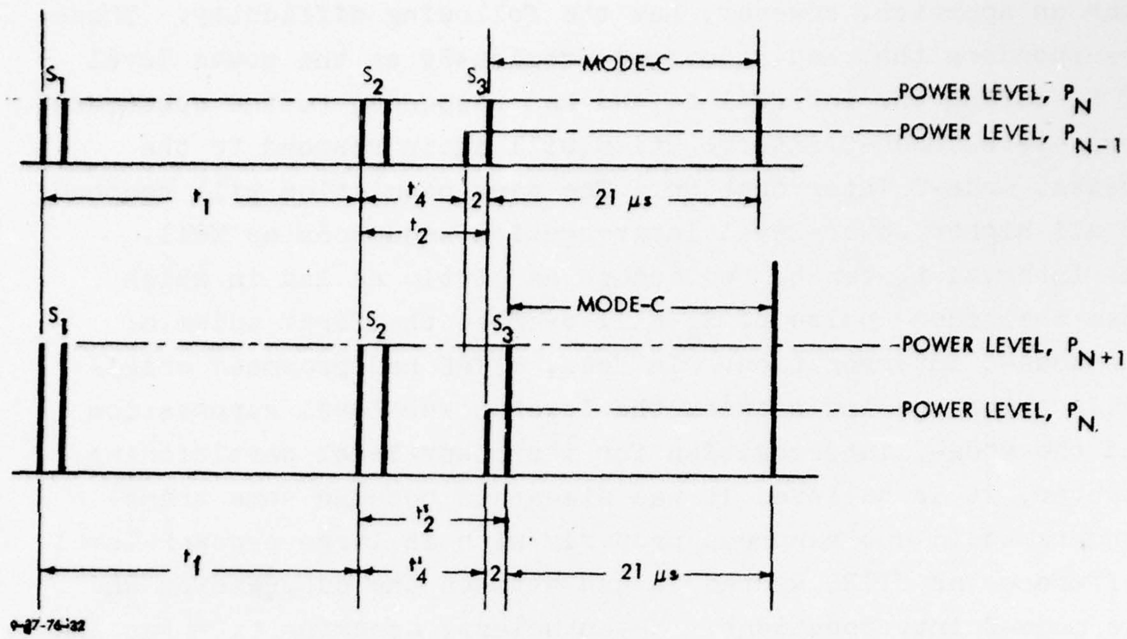
The first suppression (S_1) is intended to suppress the entire population in a given range cell (1.644 nmi). All transponders that have recovered from S_1 in the interval t_1 are resuppressed by S_2 and t_2 is made sufficiently short that none of the resuppressed transponders can respond to the mode-C interrogation. Also, some of the transponders that are suppressed by S_1 do not recover by the time that the first pulse of the mode-C interrogation is received and consequently, those transponders do not respond to the mode-C interrogating either. Only the transponders that are suppressed initially by S_1 and recover during t_2 can respond to the mode-C interrogation.

When one considers next the impact of the MITRE power-level partitioning, some arrangement of additional suppressions must be introduced to deny responses from transponders that have previously responded at lower power levels. One possibility would be to introduce a suppression (S_3) pair at the previous power level during the interval t_2 just prior (t_3) to the mode-C interrogation as follows:



Such an approach, however, has the following difficulty. Those transponders that had recovered previously at the power level of S_3 during the interval t_3 and had responded to the subsequent lower-level mode-C interrogation will again respond to the present mode-C interrogation. The same population will respond to all higher power-level interrogation sequences as well. The interval t_3 can be reduced to as little as 2μs in which case the second pulse of S_3 will overlap the first pulse of the mode-C interrogation. In fact, MITRE had proposed originally a 2μs spacing between the lower power-level suppression and the mode-C interrogation for its power-level partitioning. However, it is believed it was discarded because some transponders would not suppress properly with as large a power-level difference as MITRE wanted to use between the suppression and the mode-C interrogations. Nevertheless, assuming t_3 = 2μs and t_2 has the values suggested above (29, 4, 5 and 20μs), then the efficacy of the power-level partitioning will be reduced considerably, particularly for t_2 = 4 and 5μs. The worst case is for t_2 = 4μs. If the power level partitioning resulted in four equal-size groups in a given range-garble cell then at the highest power level, in addition to those that should respond, half of those that responded to the previous power levels would also respond. The result would be an increased number of replies by a factor of 4.5/8 = 2.5 or an efficiency of power partitioning of only a factor of 1.6.

Another approach that is even less satisfactory would be to change the resuppression delays at each power level. In this approach, the interval t_4' would be made the same as the interval t_2 of the previous power level as follows:



Such a scheme does not overcome the objection of the previous scheme and has an additional drawback. Even for $t_3 = 2$, if for the lowest power level one of the t_2's - 4 or 5μs, the highest power level t_2's will be greater by 6μs. In the worst case, if, at the lowest power level, $t_2 = 4$ and 5μs and each included 0.25 of the population, the new $t_2 = 10$ would include more than 0.5 of the population. In the light of the above discussion, the numbers of aircraft in a given garble-range cell were recalculated for the first method suggested for combining power partitioning and resuppression. The results are given in Table F-4. Comparison with the results of the previous Table F-3 show that the maximum number of aircraft in a subgroup has increased from 4.7 to 6.6 and that in more than half of the tabulated groups the number equals or exceeds 6.0.

TABLE F-4. AVERAGE NUMBER OF AIRCRAFT IN EACH SUBPOPULATION ASSUMING FIXED RESUPPRESSION DELAYS AND 2μs LOWER-POWER-LEVEL SUPPRESSION DELAYS.*

Group (i)	P = 6dbw	12dbw	18dbw	30dbw	Total A/C Incremental Population
1	0.99(2.5)	.51(1.3)	.50(1.3)	.50(1.3)	10
2	0.84(4.6)	.56(3.1)	.51(2.8)	.50(2.8)	22
3	0.61(4.7)	.60(4.6)	.54(4.2)	.51(3.9)	31
4	0.43(4.0)	.585(5.4)	.57(5.3)	.52(4.8)	37
5	0.30(2.9)	.53(5.2)	.58(5.7)	.54(5.3)	39
6	0.20(2.1)	.46(4.7)	.59(6.0)	.57(5.8)	41
7	0.14(1.4)	.41(4.2)	.59(6.0)	.59(6.0)	41
8	0.10(1.0)	.35(3.6)	.57(5.8)	.62(6.3)	41
9	0.07(0.7)	.29(2.9)	.54(5.4)	.65(6.5)	40
10	0.05(0.5)	.24(2.3)	.51(5.0)	.68(6.6)	39
11	0.035(0.3)	.20(1.8)	.48(4.4)	.71(6.5)	37
12	0.025(0.2)	.16(1.4)	.44(3.7)	.74(6.3)	34

*Worst case for resuppression partitioning corresponding to the population with the 29-33μs delay in which population with 31-33μs delay responds at higher power levels.

APPENDIX G

THE ALPHA-BETA TRACKER

The alpha-beta tracker is a computer algorithm which, on the basis of range measurements, provides smoothed estimates and predictions of range and range rate. The tracker equations are as follows:

$$\hat{r}_n = r_{np} + \alpha(r_n - r_{np})$$

$$\hat{\dot{r}}_n = \dot{r}_{np} + \frac{\beta}{T}(r_n - r_{np})$$

$$r_{np} = \hat{r}_{n-1} + T\,\hat{\dot{r}}_{n-1} \qquad \text{(G-1a, b, c, d)}$$

$$\dot{r}_{np} = \hat{\dot{r}}_{n-1}$$

where $\hat{r}_n$ and $\hat{\dot{r}}_n$ are the estimated range and range rate for the nth sample after the range measurements, r_n, is obtained; r_{np} and $\dot{r}_{np}$ are the predicted range and range rate for the nth sample before the range measurement r_n was obtained; T is the time between samples (= 1 sec for MCAS); and α and β are dimensionless tracker parameters.

Application of z-transforms* to Eqs. G-1 yields

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*See, for example, Chapter 4 of "Sampled Data Control Systems" by Ragazzini and Franklin.

$$\hat{R} = R_p + \alpha(R-R_p)$$

$$\hat{\dot{R}} = \dot{R}_p + \frac{\beta}{T}(R-R_p)$$

(G-2a, b, c, d)

$$R_p = z^{-1}\hat{R} + T z^{-1}\hat{\dot{R}}$$

$$\dot{R}_p = z^{-1}\hat{\dot{R}}$$

where $\hat{R}$, R_p, R, $\dot{R}_p$ and $\hat{\dot{R}}$ are the z-transforms of $\hat{r}_n$, r_{np}, r_n, $\dot{r}_{np}$, and $\hat{\dot{r}}_n$ respectively.

Equation G-2d is substituted into Eq. G-2b and the result is solved for $\hat{\dot{R}}$:

$$\hat{\dot{R}} = \frac{1}{1-z^{-1}}\frac{\beta}{T}(R-R_p) \qquad \text{(G-3)}$$

Equations G-3 and G-2a are substitued into Eq. G-2c which becomes

$$R_p = z^{-1}\left[R_p + \alpha(R-R_p)\right] + \frac{z^{-1}}{1-z^{-1}}\beta(R-R_p) \qquad \text{(G-4)}$$

Solving for R_p and rearranging terms yields

$$R_p = \left[\frac{(\alpha+\beta)z^{-1} - \alpha z^{-2}}{1 - (2-\alpha-\beta)z^{-1} + (1-\alpha)z^{-2}}\right] R \qquad \text{(G-5)}$$

The tracker's range gate is centered on the predicted range, r_{np}, and the gate must be wide enough to accommodate the difference between the measured range, r_n, and the predicted range r_{np}. Thus, the quantity of interest is $R - R_p$ which follows from Eq. G-5:

$$R - R_p = \left[1 - \frac{(\alpha+\beta)z^{-1} - \alpha z^{-2}}{1 - (2-\alpha-\beta)z^{-1} + (1-\alpha)z^{-2}}\right] R$$

$$= \left[\frac{(1-z^{-1})^2}{1 - (2-\alpha-\beta)z^{-1} + (1-\alpha)z^{-2}}\right] R \qquad \text{(G-6)}$$

The range measurement, r_n, is represented by

$$r_n = r_o + Vnt + \frac{1}{2} g(nT)^2 + \varepsilon_n \qquad \text{(G-7)}$$

where T is the time between samples; nT is time, V is the range rate, g is the acceleration; r_o is the range at nT = 0; and ε_n is the range measurement error. Application of the final value theorem for z-transforms shows that r_o + VnT makes no contribution to R - R_p. The contribution of the acceleration term, whose z-transform is

$$\frac{gT^2}{2} \frac{z^{-1}(1+z^{-1})}{(1-z^{-1})^3}$$

is given by

$$(R-R_{pg}) = \lim_{z \to 1} \left[(1-z^{-1}) \frac{(1-z^{-1})^2}{1 - (2-\alpha-\beta) + (1-\alpha)z^{-1}} \frac{gT^2}{2} \frac{z^{-1}(1+z^{-1})}{(1-z^{-1})^3}\right]$$

$$= \frac{gT^2}{\beta} \qquad \text{(G-8)}$$

The contribution of the measurement error term ε_n (in Eq. G-7) is evaluated by assuming independent errors from sample-to-sample; any bias errors will cancel as far as the difference $r_n - r_{np}$ is concerned. If w_0, w_1, ..., w_n..., denotes the tracker response, $r_n - r_{np}$, due to a unit sample input, and this response

is simply the inverse z-transform of the bracketted factor in Eq. G-6, then the steady-state variance of $r_n - r_{np}$ due to measurement errors is given by

$$\overline{(r-r_p)_m^{\,2}} = \sigma^2 = \sigma_m^{\,2} \sum_{n=0}^{\infty} w_n^{\,2} \qquad \text{(G-9)}$$

where $\sigma_m^{\,2}$ is the variance of the range measurement error, and w_n is the inverse z-transform of

$$W(z) = \frac{(1-z^{-1})^2}{1-(2-\alpha-\beta)z^{-1}+(1-\alpha)z^{-2}} \qquad \text{(G-10)}$$

The theorem for the sums of squares of sample sequences (see cited reference), gives

$$\sum_{n=0}^{\infty} w_n^{\,2} = \frac{1}{2\pi j} \int_{\Gamma} W(z)W(z^{-1})z^{-1}\, dz \qquad \text{(G-11)}$$

where $j = \sqrt{-1}$, and Γ is the unit circle contour of integration in the complex z-plane. For this calculation, it is convenient to express Eq. G-10 in the form

$$W(z) = \frac{(1-z^{-1})^2}{(1-az^{-1})\,(1-bz^{-1})} = \frac{(1-z)^2}{(z-a)\,(z-b)} \qquad \text{(G-12)}$$

where

$$2-\alpha-\beta = a+b \qquad \text{(G-13)}$$

and

$$1-\alpha = ab \qquad \text{(G-14)}$$

so that

$$W(z)W(z^{-1})z^{-1} = \frac{1}{z} \; \frac{(1-z)^2(1-z^{-1})^2}{(z-a)(z-b)(z^{-1}-a)(z^{-1}-b)}$$

$$= \frac{1}{z} \; \frac{(1-z)^4}{(z-a)(z-b)(1-az)(1-bz)} \qquad \text{(G-15)}$$

where the last step follows from multiplying numerator and demoninator by z^2. The poles of Eq. G-15 are 0, a, b, a^{-1}, and b^{-1} However, for a stable tracker, $|a| < 1$ and $|b| < 1$ so that the only poles inside the integration contour in Eq. G-11 are 0, a, and b. The contribution of these three poles to the contour integral in Eq. G-11 are

$$\sum w_n^{\,2} = \frac{1}{ab} + \frac{(1-a)^4}{a(a-b)(1-a^2)(1-ab)} + \frac{(1-b)^4}{b(b-a)(1-ab)(1-b^2)} \qquad \text{(G-16)}$$

where the first, second and third terms represent, respectively, the contributions from the poles at 0, a, and b. After some manipulation on the right-hand side, and after replacement of the left-hand side by $\sigma^2/\sigma_m^{\,2}$ (see Eq. G-9), we obtain

$$\frac{\sigma^2}{\sigma_m^{\,2}} = \frac{1}{ab} + \frac{b(1+b)(1-a)^3 - a(1+a)(1-b)^3}{ab(a-b)(1+a)(1+b)(1-ab)} \qquad \text{(G-17)}$$

The numerator of the second term is rewritten as follows:

$$b(1+b)(1-a)^3 - a(1+a)(1-b)^3 = b(1+b)(1-a)(1-a)^2 - a(1+a)(1-b)(1-b)^2$$

$$= b[(1-ab) - (a-b)](1-a)^2 - a[(1-ab) + (a-b)](1-b)^2$$

$$= (1-ab)[b(1-a)^2 - a(1-b)^2] - (a-b)[b(1-a)^2 + a(1-b)^2]$$

$$= (1-ab)[b+a^2b - a - ab^2] - (a-b)[b(1-a)^2 + a(1-b)^2]$$

$$= (1-ab)[(b-a) + ab(a-b)] - (a-b)[b(1-a)^2 + a(1-b)^2]$$

$$= (a-b)[(1-ab)(ab-1) - b(1-a)^2 - a(1-b)^2]$$

$$= (a-b)[-(1-ab)^2 - b + 2ab - ba^2 - a + 2ab - ab^2]$$

$$= (a-b)[-(1-ab)^2 - (a+b) + 4ab - ab(a+b)]$$

$$= (a-b)[-\alpha^2 - (2-\alpha-\beta) + 4(1-\alpha) - (1-\alpha)(2-\alpha-\beta)] \quad \text{(G-18)}$$

where the last step was obtained through application of Eqs. G-13 and G-14. Collecting terms on the right-hand side and substituting into the numerator of the second term on the right-hand side of Eq. G-17, we get, after cancellation of (a-b):

$$\frac{\sigma^2}{\sigma_m^2} = \frac{1}{ab} + \frac{1}{ab} \; \frac{-2\alpha^2 + 2\beta - \alpha\beta}{(1+a)(1+b)(1-ab)} \quad \text{(G-19)}$$

The remaining combination of a and b are expressed in terms of Eqs. G-13 and G-14; and in particular

$$\begin{aligned}(1+a)(1+b) &= 1 + (a+b) + ab \\ &= 1 + 2 - \alpha - \beta + 1 - \alpha \\ &= 4 - 2\alpha - \beta\end{aligned} \quad \text{(G-20)}$$

so that Eq. G-19 becomes

$$\frac{\sigma^2}{\sigma_m^2} = \frac{1}{1-\alpha} + \frac{1}{1-\alpha}\,\frac{-2\alpha^2 + 2\beta - \alpha\beta}{(4-2\alpha-\beta)\alpha}$$

$$= \frac{4\alpha - 2\alpha^2 - \alpha\beta - 2\alpha^2 + 2\beta - \alpha\beta}{\alpha(1-\alpha)(4-2\alpha-\beta)}$$

$$= \frac{4\alpha - 4\alpha^2 - 2\alpha\beta + 2\beta}{\alpha(1-\alpha)(4-2\alpha-\beta)}$$

$$= \frac{4\alpha(1-\alpha) + 2\beta(1-\alpha)}{\alpha(1-\alpha)(4-2\alpha-\beta)}$$

$$= \frac{4\alpha + 2\beta}{\alpha(4-2\alpha-\beta)} \tag{G-21}$$

In view of the definition of σ^2 given by the first part of Eq. G-9 we have

$$\overline{(r-r_p)_m^{\;2}} = \left[\frac{4\alpha+2\beta}{\alpha(4-2\alpha-\beta)}\right]\sigma_m^{\;2} \tag{G-22}$$

The variance, $\sigma_p^{\;2}$, of the predicted range, r_p, due to random measurement errors can be obtained by the same methods (in this evaluation, the bracketted factor in Eq. G-5 would be used in place of W(z) in Eq. G-11). The result is

$$\sigma_p^{\;2} = \left[\frac{4\alpha+2\beta}{\alpha(4-2\alpha-\beta)} - 1\right]\sigma_m^{\;2} \tag{G-23}$$

Equations G-22 and G-23 differ only by σ_m^2 because the error in the range measurement in any sample is independent of the prediction error which depends only on prior range measurements.

Both Eqs. G-22 and G-23 can be minimized with respect to α without affecting the steady-state acceleration error given

by Eq. G-8. The value of α minimizing both equations (i.e., G-22 and G-23) is the same and is obtained by the usual differentiation techniques. The optimum α is

$$\alpha_{opt} = \sqrt{\beta} - \frac{1}{2}\beta \qquad \text{(G-24)}$$

which after substitution into Eqs. G-22 and G-23 and after some manipulations, yields:

$$\overline{(r-r_p)_m{}^2} = \frac{\sigma_m{}^2}{[1-\frac{1}{2}\sqrt{\beta}]^2} = \sigma^2 \text{ for } \alpha = \alpha_{opt} \qquad \text{(G-25)}$$

and

$$\sigma_p{}^2 = \left[\frac{1}{(1-\frac{1}{2}\sqrt{\beta})^2} - 1\right]\sigma_m{}^2 \text{ for } \alpha = \alpha_{opt} \qquad \text{(G-26)}$$

Equations G-26 and G-8 exhibit the usual behavior of tracking systems: as $\beta \rightarrow 0$ the effective smoothing time of the tracking loop increases to infinity and the prediction error, due to random measurement errors, approaches zero while the prediction error component due to any acceleration, approaches infinity.

The range measurement error includes contributions from pulse jitter, clock quantization errors and garble induced errors. The pulse jitter is assumed to be uniformly distributed between the specified* limits, ± 0.1 μsec, while the clock quantization errors are uniformly distributed between ± 0.0604 μsec, corresponding to a clock period of 8.276 MHz. Consequently,

$$\text{One-sigma measurement error without garble} = \frac{1}{\sqrt{3}}\sqrt{(0.1)^2 + (0.0604)^2} \times 492 \text{ ft/μsec}$$

$$= 33.2 \text{ ft} \qquad \text{(G-27)}$$

*DOT/FAA standard for ATCRBS.

Garble brackets occurring within the range window (±240 ft) will contribute an error between ±240 ft, provided that the garble amplitude is greater than that of the signal; otherwise, the error contribution is assumed to be zero. Thus, the conditional variance of the range measurement error, given a dominant interfering (within range window) garble bracket, is $(240)^2/3$ ft^2.

The probability of one or more interfering and dominant brackets within ±240 ft is represented by

$$1 - e^{-B}$$

in which B is the expected number of dominating interfering brackets. It is assumed that the interfering bracket is equally likely to be stronger or weaker than the signal so that B is simply one-half of the number of interfering brackets as computed in Eq. C-18 of Appendix C; provided that N is replaced by N-1 to account for the fact that one of the N overlapping replies is the desired reply and that only N-1 are contributing garble. In other words:

$$B = 0.488 \left\{ \frac{5(N-1)}{20.3} \left[1 - (0.877)^{N-1}\right] + \frac{N-1}{20.3} \right\} \qquad \text{(G-28)}$$

The overall range measurement error variance, σ_m^2, becomes

$$\sigma_m^2(\text{ft}^2) = (33.2)^2\, e^{-B} + \frac{(240)^2}{3}\left(1 - e^{-B}\right) \qquad \text{(G-29)}$$

where $(33.2)^2$ is the variance, given no dominant interfering garble brackets, and $240^2/3$ is the variance given that the range gate was hit by a strong interfering garble bracket, while e^{-B}, and $1 - e^{-B}$ are the probabilities for the two events.

Substituting Eq. G-28 into Eq. G-29 we obtain

$$\sigma_m = \begin{cases} 53.7 \text{ ft} & N=3 \\ 65.0 \text{ ft} & N=4 \\ 75.4 \text{ ft} & N=5 \end{cases} \qquad \text{(G-30)}$$

which, when substituted into Eq. G-25, gives the variance, σ^2, of the error between the predicted range r_p and the measured range r. The probability density function of this error is sketched in Fig. G-1. Thus, to the extent that the distribution can be approximated by a Gaussian, the probability of losing a range measurement (and hence the complete reply) is minimized by maximizing the ratio

$$\rho = \frac{240 - (gT^2/\beta)}{\sigma} = \frac{1}{\sigma_m} [240-(gT^2/\beta)](1 - \tfrac{1}{2} \sqrt{\beta}) \qquad \text{(G-31)}$$

where the right-hand side is obtained with the help of Eq. G-25. Equation G-31 can be interpreted as the normalized tracking margin. This is the difference between the mean tracking error, gT^2/β, and the gate edge, at 240 ft, normalized by the one-sigma tracking error. The values of Eq. G-31 as a function of β (with $g = 32$ ft/sec^2, $T = 1$ sec) are:

β	Eq. G-31
0.4	$109.4/\sigma_m$
0.5	$113.8/\sigma_m$
0.6	$114.4/\sigma_m$
0.7	$113.0/\sigma_m$
0.8	$110.6/\sigma_m$

Under these conditions (i.e., $g = 32$ ft/sec^2 and $T = 1$ sec),

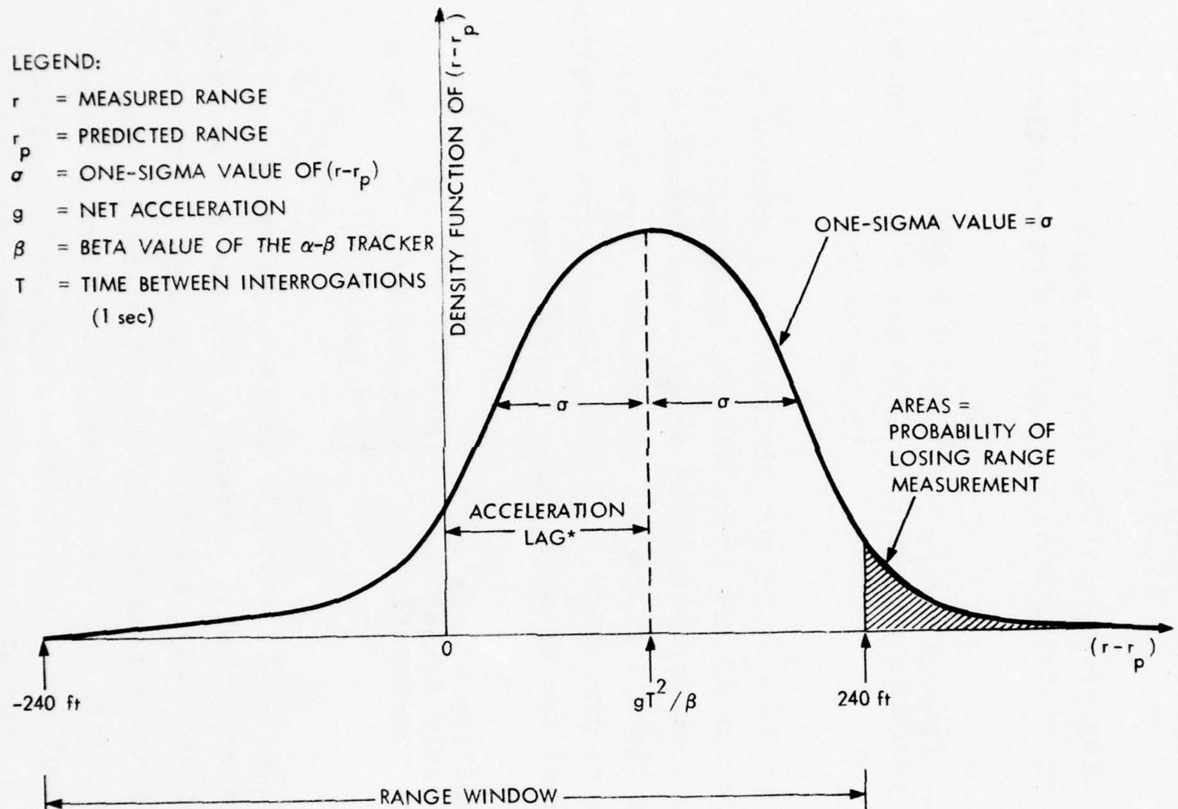
$$\beta_{opt} = 0.6 \qquad \text{(G-32)}$$

is the optimum choice of β, which, together with Eq. G-24, gives

$$\alpha_{opt} = 0.4746 \qquad \text{(G-33)}$$

as the optimum α. The corresponding maximum of Eq. G-31 is

$$\rho_{max} = \frac{114.4 \text{ (ft)}}{\sigma_m \text{ (ft)}} \qquad \text{(G-34a)}$$



*Obtained from Eq. G-8.

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FIGURE G-1. Sketch of Density Function of the Difference Between Measured Range, r, and Predicted Range r_p

or (by using Eq. G-30),

$$\rho_{max} = \begin{cases} 2.13 \text{ for } N=3 \\ 1.76 \text{ for } N=4 \\ 1.52 \text{ for } N=5 \end{cases} \qquad \text{(G-34b)}$$

The corresponding probabilities that the reply is lost (again, assuming a Gaussian distribution) on a given interrogation, is

$$\text{Probability of reply loss per interrogation} = \begin{cases} 0.02 \text{ for } N=3 \\ 0.04 \text{ for } N=4 \\ 0.06 \text{ for } N=5 \end{cases} \qquad \text{(G-35)}$$

If the acceleration (0.5g per aircraft as specified by ANTC-117) persists for 10 sec there will be essentially five independent reply loss opportunities because the effective tracker smoothing time, for the α-β parameters given by Eqs. G-32 and G-33, is of the order of 2 sec. Under these conditions, the probabilities of one or more reply losses become:

$$\text{Probability of one or more reply losses for 10 sec acceleration} = \begin{cases} 0.1 \text{ for } N=3 \\ 0.2 \text{ for } N=4 \\ 0.3 \text{ for } N=5 \end{cases} \qquad \text{(G-36)}$$

Given a reply loss, the track is coasted, in which case the tracker prediction errors will increase. For the α-β parameters given by Eqs. G-32 and G-33, and for one lost reply, both the one-sigma prediction error as well as the acceleration lag error will each increase 2.1 times (this can be demonstrated by the application of the z-transform methods used to obtain Eqs. G-8 and G-22). For all practical purposes, this leads to track loss if the tracking gate is not increased after the first reply loss.

If the reply loss is detected and if the range window (i.e., range gate) is doubled to insure track reacquisition, then,

inevitably, the same logic would be operative during a false (i.e., phantom) track. The result is an increase in false track survival probability and, hence, an increase in the false alarm rates computed in Appendix C where a constant range window, ±240 ft, was employed.

Another factor affecting tracking reliability is the tracker response in the presence of a crossing aircraft which, for two or more interrogations, remains within the range window of the intruder under track. If the crossing aircraft provides the dominant return, the probability of which is assumed to be 0.5, and if the crossing aircraft range is within the gate half-width, 240 ft ($\approx$ one half the pulse width), then the range measurements will be determined by the range of the crossing aircraft. The resultant tracker response is obtained as follows:

1. Let s denote the range difference between the crossing aircraft and the reference aircraft, i.e., the intruder under track, at the first interrogation that the crossing aircraft falls within the range gate.
2. Let δ denote the change in the range difference (between the crossing and reference aircraft) between successive interrogations. Since the interrogations are spaced at 1 sec intervals, δ (ft) is also the range rate difference in ft/sec.
3. Assume that before the crossing aircraft entered the range window, the α-β tracker had a perfect range track on the nonaccelerating intruder. In other words, the tracking errors examined earlier are neglected, and the tracker has reached a steady state where the tracking error is zero for a nonaccelerating intruder.
4. Because of (3), the basic tracker equations, i.e., Eq. G-1, can be applied directly by replacing all ranges and range rates by range differences and range-rate differences between the crossing aircraft and the reference aircraft.

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Thus, in view of (1), (2), (3), and (4), the tracker input is the series of range measurements: s, $s+\delta$, $s+2\delta$, $s+3\delta$,...etc., starting with the first reply obtained after the crossing aircraft entered the range window. Consequently, the measurements consist of a "step" function of amplitude, s, plus a "ramp" of slope δ. The z-transform for the latter is

$$\delta \frac{z^{-1}}{(1-z^{-1})^2}$$

which, when substituted for R on the right-hand side of Eq. G-6 gives the z-transform:

$$\frac{z^{-1}}{1-(2-\alpha-\beta)z^{-1}+(1-\alpha)z^{-2}}\,\delta$$

of the tracker response to the "ramp". Addition of the response from the "step", to be completed below, gives the complete tracker response to the disturbance from a crossing aircraft. The inverse z-transform* of the above gives:

$$\begin{array}{l}\text{response to}\\ \text{range ramp}\end{array} = \delta\,\frac{h^n \sin nq}{h \sin q} \qquad \text{(G-37)}$$

where $n = 0, 1, 2, \ldots$, denotes the reply number starting with $n=0$ for the first reply from the crossing aircraft inside the range window; and where

$$h = \sqrt{1-\alpha} = 0.7248 \qquad \text{(G-38)}$$

$$q = \cos^{-1}\frac{2-\alpha-\beta}{2\sqrt{1-\alpha}} = 50.33^{\circ} \qquad \text{(G-39)}$$

in which the numerical values were obtained by using Eqs. G-32 and G-33.

---

* See, for example, the reference cited earlier in this Appendix.

Z-transform techniques also yield the response to the "step":

$$\frac{s}{h \sin q} [h^{n+1} \sin(n+1)q - h^n \sin nq]$$

which, when added to Eq. G-37 and after rearrangement of terms, gives:

$$\text{tracker response} = \frac{h^n}{h \sin q} [s\, h \sin(n+1)q - (s-\delta)\sin nq] \quad \text{(G-40)}$$

If the closing range rate (with respect to the interrogator) of the crossing aircraft is smaller than that of the intruder and if the magnitude of this difference is less than 240 ft/sec then:

$$0 \le \delta \le 240 \text{ ft} \quad \text{(G-41a)}$$

and

$$-240 \text{ ft} \le s \le -240 \text{ ft} + \delta \le 0 \quad \text{(G-41b)}$$

where Eq. G-41b simply states that the first range measurement on the crossing aircraft must lie somewhere between the gate edge, s = 240 ft, and δ (ft) inside the gate, or $s = -240 + \delta$.

Thus, the expected value of s is

$$\text{average } s = -240 + (\delta/2) \quad \text{(G-42)}$$

in which case, Eq. G-40 becomes

$$\text{expected response} = \frac{h^n}{h \sin q} [(-240+\tfrac{\delta}{2})\, h \sin(n+1)q + (240+\tfrac{\delta}{2}) \sin nq] \quad \text{(G-43)}$$

Tracker overshoot, which is the maximum magnitude of Eq. G-43 (as a function of n), will not exceed 240 ft (= gate half-width) as long as δ remains less than 155 ft. Thus, as long

as the overshoot, which is defined relative to the range of the crossing aircraft, remains less than 240 ft, the tracker will be pulled off by the crossing aircraft so that

$$\begin{array}{l}\text{Probability of}\\ \text{pull-off by a}\\ \text{crossing aircraft}\end{array} = \frac{1}{2}\,\mathrm{Pr}[\delta \le 155\ \mathrm{ft}] \quad \text{for } 0 \le \delta \le 240\ \mathrm{ft} \qquad \text{(G-44a)}$$

where 1/2 is the probability that the reply from the crossing aircraft dominates that of the intruder.

The preceding analysis assumes that the track reverts to the legitimate intruder as soon as a single reply from the crossing aircraft falls outside the tracking gate (i.e., overshoot exceeds 240 ft). For this reason, the analysis underestimates the pull-off probability. However, the result does provide an approximate pull-off probability when the time that the crossing aircraft remains within ±240 ft of the intruder is comparable to, or less than, the tracker settling time (to be discussed later in this analysis).

The distribution of δ (ft), appearing in the above relations, is the same as the distribution of range-rate differences, in ft/sec. Inasmuch as range rates can be approximated* by a truncated Gaussian, with a one-sigma value of 325 ft/sec, the distribution of range-rate differences, and hence δ, follows an approximate Gaussian with a one-sigma value of $325\sqrt{2} = 460$ ft. Application of this distribution to Eq. G-44, together with its symmetric counterpart for negative δ (for crossing aircraft which overtake the intruder), gives

$$\begin{array}{l}\text{Probability of}\\ \text{pull-off by a}\\ \text{crossing aircraft}\end{array} = 0.136 \qquad \text{(G-44b)}$$

---

*See IDA Study S-424 "A Review and Analysis of the Honeywell Collision Avoidance System", Vol. II, Appendix K, FAA-RD-73-151, II, October 1973.

During the 30 sec interval of track establishment, the intruder range is expected to close by 1.28 nmi, corresponding to an expected closing rate of 260 ft/sec. Thus, the expected number of crossing aircraft encountered by the intruder during 30 sec is

$$1.28 \times \frac{N-1}{1.64} = 0.780\ (N-1) \qquad (G-45)$$

where N is the number of overlapping replies (as defined in Section IV-C), or the number of aircraft per 1.64 nmi range increment, and N-1 is that number exclusive of the intruder. Thus, the average number of pull-off events is the product of Eqs. G-44b and G-45:

$$\text{Expected number of pull-offs in 30 sec} = 0.106\ (N-1) \qquad (G-46)$$

The impact of a pull-off depends on whether or not the event can be recognized by the data processing logic of the MCAS. If the pull-off is recognized, then the tracking gate can be widened in an attempt to reacquire the intruder. Otherwise, the intruder track is lost and 30 sec or more will elapse before the track can be reestablished.

The probability that the pull-off is not recognized is the probability that the altitude data of the crossing aircraft will pass the reply acceptance criteria embodied in Eqs. C-19, C-20 and C-21 of Appendix C. Evaluation of this probability follows the methodology employed in Appendix C. The convenient starting point is Tables C-3 and C-4 of Appendix C, where the garble C-bit sequences are replaced by the legitimate C-bit sequences that would be used by the crossing aircraft. These are sequences (b), (d), (e), (f) and (g). The predicted triplet used for the intruder in any one of the five sequences identified across the top row of Table C-4, Appendix C shows that:

(1) Equation C-19 of Appendix C has no impact in identifying a pull-off event because the sequence (h) will never be used in the altitude code; and

(2) The probability that the reply from the crossing aircraft is not rejected by Eq. C-20 of Appendix C is,

$\frac{4}{5}$ for TR1, 2, 4, and 5

1 for TR3

because the crossing aircraft is equally likely to transmit any one of the sequences (b), (d), (e), (f) and (g).

Since any one of the five triplets (identified across the top row of Table C-4 in Appendix C) is equally likely to be used for intruder tracking, the overall probability that the crossing aircraft is not rejected by the logic of Eqs. C-19 and C-20, of Appendix C, is given by

$$\frac{1}{5} \bullet \frac{4}{5} \times 4 + \frac{1}{5} \bullet 1 = 0.840 \qquad \text{(G-47)}$$

Since the bulk of the aircraft population is below 15,000 ft, both the intruder and the crossing aircraft will transmit, in addition to C-bits, some combination of the five bits: A_2, A_4, B_1, B_2, B_4 (see Fig. 2, Section III). If the aircraft are assumed to be uniformly distributed* between 0 and 15,000 ft, then the probability that an aircraft transmits a binary "one" in any one of the five bit positions is

$$\frac{0.5}{1-(\frac{1}{2})^5} \approx 0.5, \qquad \text{(G-47a)}$$

---

*Any tendency to bunch up in narrower altitude bands can be shown (by repeating the subsequent analysis) to decrease the probability that a pull-off is recognized.

a result which follows from the properties of the altitude code, and the assumed altitude distribution. Thus, if x denotes the number of binary "ones" of the intruder code (among the A_2, A_4, B_1, B_2, B_4 bit positions), then the probability that the crossing aircraft has at least one binary "one" in common with the intruder is

$$1 - (1-0.5)^x = 1 - (0.5)^x, \qquad \text{(G-48)}$$

a probability which is conditioned on x. The distribution of x is a Binominal distribution, conditioned on the fact that $x \geq 1$, in other words:

$$\frac{1}{1-(0.5)^5} C_x^5 (0.5)^x (1-0.5)^{5-x} .$$

When Eq. 48 is weighted by this distribution we obtain the unconditional probability that the intruder and crossing aircraft have at least one common binary "one" among the A/B bits:

$$\sum_{x=1}^{5} [1-(0.5)^x] \frac{1}{1-(0.5)^5} C_x^5 (0.5)^x (1-.05)^{5-x} = 0.7873 \qquad \text{(G-48a)}$$

The efficacy of the correlation test (Eq. C-21 of Appendix C) is evaluated in a manner similar to that used in deriving Table C-6 of Appendix C. First, the C-bit contribution to decorrelation is obtained from inspection of Table C-4 of Appendix C. For example, when the triplet TR1 is used for predicting intruder altitude then the decorrelation numbers, for unrejected legal C-bit sequences, (b), (d), (f) and (g), are respectively: 0, 4/3, 1/3, 0. Since each of these is equally likely, given that sequence (e) was rejected, (and this effect was accounted for in the analysis leading up to Eq. G-47) the expected decorrelation for TR1 is

$$\frac{1}{4} [0 + \frac{4}{3} + \frac{1}{3} + 0] = \frac{5}{12}$$

Similarly, we obtain 1/4, 2/15, 1/4, and 5/12, for TR2, 3, 4, and 5, respectively. Inasmuch as each of the five triplets is equally likely to be used for prediction, we obtain:

$$\text{Average decorrelation contributed by C-bits} = \frac{1}{5}\left[\frac{5}{12} + \frac{1}{4} + \frac{2}{15} + \frac{1}{4} + \frac{5}{12}\right] = 0.2933 \qquad \text{(G-49)}$$

Contributions from other bits (A_2, A_4, B_1, B_2, B_4), given at least one common binary "one" between the intruder and crossing aircraft (and this effect was accounted for in the analysis leading up to Eq. G-48), are obtained as follows:

1. Let x denote the number of "ones" in the prediction so that (5-x) is the number of "zeros".
2. Given x, the average number of unexpected "ones" (as defined below Eq. C-21 of Appendix C) is

   $$(5-x) \times 0.5$$

   where 0.5 is, approximately, the probability that the crossing aircraft has a binary "one" in a given position (see Eq. G-47a and related discussion).
3. Similarly, given x, the average number of unexpected "zeros" is

   $$0.5x$$

Using (2) and (3), the decorrelation number (refer to Eq. C-24 of Appendix C) becomes

$$(5-x) \times 0.5 + \frac{1}{3} \cdot 0.5\, x = 2.5 - \frac{1}{3}\, x \qquad \text{(G-50)}$$

which is a conditional average decorrelation, the condition being that there are x "ones" among the A_2, A_4, B_1, B_2, and B_4 bits of the intruder code. Since the average x is approximately 5/2, the average value of Eq. G-50 gives:

$$\text{Average decorrelation contributed by A/B bits} = 1.67 \qquad \text{(G-50a)}$$

Using Eqs. G-49 and G-50 in Eq. C-27 of Appendix C, we obtain

$$\text{Average correlation number} = 48 - 3\,(1.67+0.29) = 42.1 \qquad \text{(G-51)}$$

Reference to Table C-6 of Appendix C shows that this value is higher than the average correlation number obtained for a legitimate lightweight code (containing two binary "ones") corrupted by garble. Since the correlation threshold must be low enough to insure that such replies are not lost, the expected correlation number between an intruder and a crossing aircraft, as calculated by Eq. G-51, will, inadvertently, end up above the *correlation threshold*. Thus, the only available logic for recognizing a pull-off is that discussed in connection with Eqs. G-47 and G-48a so that

$$\begin{array}{l}\text{Conditional probability}\\ \text{of an undetected pull-off} = 0.840 \times 0.7873 = 0.661,\end{array} \qquad \text{(G-52)}$$

the condition being that a crossing aircraft with a dominant signal has entered the intruder's gate. The expected number of undetected pull-offs is the product of (1) probability of pull-off by a crossing aircraft, as given by Eq. G-44b, (2) expected number, given by Eq. G-45, of crossing aircraft encountered during 30 sec (= time for track establishment); and (3) Eq. G-52. Hence,

$$\begin{array}{l}\text{Expected number of}\\ \text{undetected pull-offs}\\ \text{during 30 sec} = 0.0701\,(N-1).\end{array} \qquad \text{(G-53)}$$

Assuming a Poisson relation between the mean number of events and the probability of one or more events, we obtain

$$\begin{array}{l}\text{Probability of one or more}\\ \text{undetected pull-offs}\\ \text{during 30 sec}\end{array} = 1 - e^{-0.0701(N-1)}$$

$$= \begin{cases} 0.13 \text{ for } N=3 \\ 0.19 \text{ for } N=4 \\ 0.24 \text{ for } N=5\,. \end{cases} \qquad \text{(G-54)}$$

These probabilities can be decreased by sacrificing tracking performance for accelerating aircraft encounters. For example, if β is reduced to 0.05 and α reduced to 0.1348 (one of several α-β value pairs used by MITRE; however, no final choice has yet been made) then the tracker's acceleration lag becomes (refer to Eq. G-8) 640 ft at $g = 32$ ft/sec^2 or 0.5g per aircraft as specified by ANTC-117. This places the intruder's reply 640 ft - 240 ft = 400 ft beyond the gate edge and the probability of track disruption is 100 percent. However, the undetected pull-off probability (without acceleration) by a crossing aircraft is decreased as follows:

(1) For $\alpha = 0.1348$ and $\beta = 0.05$, Eqs. G-38 and G-39 are replaced by $h = 0.9302$ and $q = 12.645$ deg.

(2) The maximum tracker response, Eq. G-43, will not exceed 240 ft as long as $\delta \leq 61.1$ ft. Accordingly, Eq. G-44a is replaced by $0.5 \Pr[\delta \leq 61.1 \text{ ft}]$.

(3) Eq. G-44b is replaced by 0.053 so that Eq. G-53 becomes $0.027(N-1)$ and Eq. G-54 is reduced to

$$\begin{array}{l}\text{Probability of one or}\\ \text{more undetected pull-}\\ \text{offs in 30 sec}\end{array} = \begin{cases} 0.053 & \text{for } N=3 \\ 0.078 & \text{for } N=4 \\ 0.10 & \text{for } N=5\ . \end{cases} \qquad \text{(G-55)}$$

Further analyses did not reveal any α-β pairs which could ensure tracking under accelerating encounters while maintaining a low probability of undetected pull-offs. For example, if β is set equal to 0.2 then the tracker's acceleration lag (refer to Eq. G-8 for $g = 32$ ft/sec^2 and $T = 1$ sec) is 160 ft which places the intruder's reply at 240 - 160 = 80 ft inside the gate edge (refer to Fig. G-1). Under these conditions, the probability of pull-off decreases as α is decreased to zero. At $\alpha = 0$ the probability of pull-off is only slightly smaller than that given by Eq. G-55, namely: 0.046, 0.068, and 0.087 for N=3, 4 and 5 respectively. However, as α approaches zero

while β is held constant, the range estimation and prediction errors approach infinity (refer to Eqs. G-22 and G-23) which disrupts tracking with or without acceleration. Furthermore, the envelope of the tracker's transient response is of the form of (refer to Eqs. G-38 and G-40)

$$h^n = (\sqrt{1-\alpha})^n \qquad \text{(G-56)}$$

which means that as $\alpha \to 0$, the tracker never reaches steady-state tracking conditions before n = 30 sec. Thus, even if β is large enough for steady-state tracking of an accelerating intruder, a small α could still prevent tracking at the onset of the aircraft turn.

In general, the tracker settling time, which is defined as the value of n where Eq. G-56 drops to e^{-1}, namely

$$\text{settling time} = -\frac{2}{\ell n(1-\alpha)} \qquad \text{(G-57)}$$

should not exceed the build-up time of the relative acceleration between interrogator and intruder. This imposes additional constraints on the α-β tracker parameters and, hence, on the attainable tracking performance in terms of pull-offs. The implications of this constraint are explored in Table 1 showing the computed values of:

(1) The maximum δ for which the maximum of the tracker response, i.e., the maximum of Eq. G-43 as a function of n, does not exceed 240 ft. This computation also involves Eqs. G-38, G-39, expressing h and q in terms of α and β, as well as Eq. G-57 which relates, α, to the tracker settling time.

(2) The one-sigma value, σ, of the difference between the range measurement and tracker range prediction as given by Eq. G-22. The value of σ_m used here is 65 ft, in accordance with Eq. G-30 for N = 4.

TABLE G-1. COMPUTED* VALUES OF δ AND σ AS A FUNCTION OF TRACKER SETTLING TIME AND β

Value of β and resultant accelerations lag	Tracker settling time and corresponding α					
	4 sec α = 0.3935		8 sec α = 0.2212		16 sec α = 0.1175	
	δ(ft)	σ(ft)	δ(ft)	σ(ft)	δ(ft)	σ(ft)
β = 0.1 lag = 320 ft	128.3	78.2	91.0	77.4	65.1	81.1
β = 0.2 lag = 160 ft	138.1	83.9	100.8	85.5	76.3	93.7
β = 0.4 lag = 80 ft	150.0	95.2	105.6	100.9	80.0	116.5

*Computed in accordance with the methods outlined below Eq. G-57.

TABLE G-2. COMPUTED* VALUES OF PROBABILITY OF RANGE TRACK DISRUPTION DUE TO ACCELERATION** PER INTERROGATION (UPPER LEFT-HAND CORNER OF EACH ENTRY), AND OF UNDETECTED PULL-OFF PROBABILITY (LOWER RIGHT-HAND CORNER) IN 30 SEC, AS A FUNCTION OF α-β AND TRACKER SETTLING TIME.

β - values and resultant acceleration lag	Tracker settling time and corresponding α		
	4 sec α = 0.3935 %	8 sec α = 0.2212 %	16 sec α = 0.1175 %
β = 0.1 lag = 320 ft	100 / 16	100 / 12	100 / 8.3
β = 0.2 lag = 160 ft	17 / 17	18 / 13	20 / 9.9
β = 0.4 lag = 80 ft	5 / 18	6 / 13	9 / 10

*Computed in accordance with the methods outlined below Eq. G-57.

**0.5g per aircraft, as specified by ANTC-117.

(3) The acceleration lag error, as given by Eq. G-8 with $T = 1$ sec and $g = 32$ ft/sec^2 (i.e., 0.5g per turning aircraft, as specified by ANTC-117). These results are used in Table G-2, to compute:

1. The probability of range track disruption due to acceleration in accordance with Fig. G-1.
2. The probability of undetected pull-off by a crossing aircraft, in accordance with the procedure described between Eq. G-43 and G-54.

Table G-2 shows that the undetected pull-off probability decreases for decreasing α-β values and increasing tracker settling time. However, such decreasing values create serious problems during accelerating encounters when the smallest α-β values (i.e., $\alpha = 0.1175$, $\beta = 0.1$), and hence the smallest undetected pull-off probability in Table G-2, are employed. In order to gain some insight into the problem, assume that:

1. Only one of the two aircraft in an encounter rolls into an 0.5g turn,
2. The turn is maintained for 12 sec which permits a 200 knot aircraft to turn only 33 deg, and
3. Roll-in and roll-out times are short compared to the duration of the turn.

Because of the tracker settling time, 16 sec, is long compared to the roll-in and roll-out times of the intruder, the tracker response can be approximated by the response to an acceleration pulse lasting 12 sec. The acceleration pulse is equivalent to two acceleration steps, in opposite directions, spaced by 12 sec. Using the standard z-transform techniques, similar to those used in deriving Eq. G-37, we obtain the response to a 12-sec acceleration pulse as

$$gT^2 \sum_{n=0}^{n=k} \frac{h^n \sin nq}{h \sin q} \quad \text{for } k \leq 12 \text{ sec}$$

$$gT^2 \sum_{n=0}^{12} \frac{h^n \sin nq}{h \sin q} - gT^2 \sum_{n=12}^{n=k} h^{(n-12)} \frac{\sin(n-12)q}{h \sin q} \quad \text{for } k \geq 12 \text{ sec}$$

where g is the acceleration and T = 1 sec is the time between samples. The results are tabulated Table G-3. In addition to the transient response error, the tracker will be subjected to random errors with a one-sigma value of 81.1 ft (see Table G-1 for β = 0.1 and settling time = 16 sec). Thus, if the gate is maintained at ±240 ft, the probability of track loss somewhere between 7 and 12 sec (see Table G-3) is essentially 50 percent because the random error will not change significantly over an interval which is short compared to the tracker's settling time.

TABLE G-3. TRACKING-ERROR TRANSIENT

Conditions: β = 0.1, α = 0.1175 (settling time = 16 sec)
Intruder acceleration = 16 ft/sec^2
Interrogator acceleration = 0 ft/sec^2
Duration of acceleration = 12 sec

TIME, sec	ERROR, ft	TIME, sec	ERROR, ft
0	0	12	218.4
1	16.0	13	181.8
2	44.5	14	131.3
3	81.2	15	73.7
4	121.5	16	15.4
5	160.9	17	-37.6
6	195.6	18	-80.6
7	222.6	19	-110.4
8	240.2	20	-125.7
9	247.7	21	-126.7
10	245.6	22	-114.9
11	235.1	23	- 92.9

If, upon loss of a reply, special logic is included to widen the gate, then the track loss probability could be decreased. For example, if the gate were increased from its normal size of 480 ft (i.e., ±240 ft) to 600 ft, and if the additional 120 ft were all applied in the direction of maximum tracking error expected from closing accelerations, then the probability that the 247.7-ft transient error (refer to Table G-3 for time = 9 sec) plus the random error, (one sigma value = 81.1 ft as indicated earlier) exceeds the increased gate limit at 240 ft + 120 ft = 360 ft is decreased from 50 percent to 8.3 percent (assuming a Gaussian distribution for the random error). Because the random errors are nearly perfectly correlated for time intervals small compared to the tracker settling time, reply losses will tend to bunch up over three to four second intervals during which time the intruder will pull away from the tracker.

If both aircraft were accelerating at 0.5g, then the track loss probability will exceed 50 percent in spite of the increased gate size, i.e., 600 ft instead of 480.

Although the increased gate size improves tracking in accelerating encounters, any logic which triggers a gate increase when no reply is received within the normal gate, would also trigger almost continuous gate increases during a false track. Under such conditions, the probability of false track survival is increased and the effect, corresponding to a 25 percent gate increase (from 480 ft to 600 ft), is equivalent to a 25 percent increase in aircraft population. Reference to Eq. C-44 of Appendix C shows that a 25 percent population increase, from N=4 to N=5, will increase the false track survival probability by

$$\frac{0.7211 \times 10^{-1}}{0.1276 . 10^{-2}} = 56.5 \text{ times} \qquad \text{(G-58)}$$

In other words, the alarm rate of 1.1 per hour (see Table 4, Section VI-C-1 of the main text, for N = 4) obtained for the normal gate size, increases to 62 alarms per hour.

In summary, two mechanisms for range track disruption have been examined in this appendix: (1) interrogator-intruder relative accelerations encountered during turns such as specified in ANTC-117 (i.e., 0.5g per aircraft); and (2) track pull-off by a crossing aircraft whose range may sweep through the range window of the intruder under track. In the first instance, the combined errors due to acceleration lag in the tracker, the range measurement errors due to garble, pulse jitter and clock quantization, may place the extrapolated range window outside the intruder range. In the second instance, the range track may be transferred, inadvertently, to a crossing aircraft whose reply signal is stronger than that of the intruder and whose range rate is close enough to that of the intruder. Furthermore, if the altitude code of the crossing aircraft is not rejected by the data processing logic then the pull-off remains undetected and the intruder track is lost. Thus, the probability of undetected pull-off during 30 sec (needed for track establishment) prior to alarm is a lower bound* on the probability of false dismissal of legitimate threats.

The basic limitations of MCAS are illustrated by three design cases. In each case the aircraft population is assumed to produce a reply overlap of N=4. This means that 49 aircraft can be within 20 nmi (=instrument BCAS range) of the interrogator, if no whisper-shout is employed, or 195 aircraft when a four-fold improvement is postulated for whisper-shout. The latter is still below the 412 aircraft projected by FAA for the Los Angeles terminal area in the 1980s. The specific cases are as follows:

*Undetected pull-off is only one mechanism producing false dismissals.

Case A. This case, shown in Table G-4, illustrates the performance attainable when the α-β parameters are optimized to minimize the probability of track disruption under accelerating encounters. This probability was thus reduced to a relatively low 4 percent, but the probability of undetected pull-off turned out to be a relatively high 19 percent. In a qualitative sense, the result is not unexpected; the range rate difference between a crossing aircraft (with a dominant signal) and the intruder acts as an apparent accleration of the intruder. Consequently, a tracker designed to follow an accelerating intruder will also tend to follow a dominant crossing aircraft.

Case B, shown in Table G-4, illustrates the problems encountered when the α-β parameter pairs are readjusted to decrease undetected pull-offs (results for other pairs are shown in Table G-2). In this case, the tracker has no capability during accelerations of the 0.5g per aircraft, specified by ANTC-117. If, on the other hand, the acceleration does not exceed 0.2g then the probability of track loss decreases to 50 percent.

Case C postulates gate-size control logic, which upon loss of a reply within the normal gate size, 480 ft, increases the gate to 600 ft. Furthermore, the additional 120 ft are all applied in the direction of the expected tracking error for closing accelerations. Also, the α-β parameters used in Case B were modified to β = 0.1 and α = 0.1175 (corresponding to a tracker settling time of 16 sec); the reasons being that (1) the increased gate size, from 480 to 600, produced no significant improvement in Case B performance during accelerating encounters, and (2) the change in α-β parameters together with the increased gate size produced some improvement in acceleration tracking with only a slight increase in pull-off probability from 7.8 percent to 8.3 percent. The resultant track loss probabilities in accelerating encounters were:

1. 8.3 percent when only one aircraft is accelerating at 0.5g.
2. 50 percent or more when both aircraft were accelerating, depending on the relative time and buildup of acceleration.

This represents a significant improvement over Case B. However, the increased gate-size control logic used in Case C impacts the false-alarm rate. Any logic, which increases the gate size upon loss of a reply, or replies, is also operative during false tracks and improves the false track survival probability. Whereas, a fixed gate size of 480 ft produced a false-alarm rate of 1.1 per hour, the use of inereased gates, although limited to 600 ft, produced 62 alarms per hour.

TABLE G-4. TRACKER PERFORMANCE AS A FUNCTION OF α-β PARAMETERS, USING A ±240 FT WINDOW

Conditions: (1) Number of overlapping replies = 4 (=number of aircraft per 1.64 nmi range increment)

(2) Aircraft population, within 20 nmi, for the above is 49 without "whisper shout", and 195 with "whisper shout".

Case A optimizes α-β parameters to minimize track disruption during acceleration.

Case B sacrifices performance during acceleration in order to reduce undetected pull-off probability (other cases are shown in Table G-2).

Tracker performance	Case A $\alpha = 0.4746$ $\beta = 0.6$	Case B $\alpha = 0.1348$ $\beta = 0.05$
Probability of range track disruption, per interrogation, during acceleration[5]	4%[1]	100%[3]
Probability of undetected pull-off during 30 sec	19%[2]	7.8%[4]

[1] From Eq. G-35 and related analyses.
[2] From Eq. G-54 and related analyses.
[3] From analyses preceding Eq. G-55.
[4] From Eq. G-55 and related analyses.
[5] 0.5 "gees" per aircraft, as specified in ANTC-117.

Conclusions: The present study did not uncover any usable α-β parameter pair which can ensure reliable tracking in accelerating encounters (0.2 to 0.5 turn g per aircraft) without a severe penalty in false dismissals due to undetected track pull-offs during non-accelerating encounters. Attempts to improve tracking in accelerating encounters, through programmed gate-size increases, produced very high false-alarm rates.

MITRE is experimenting (by computer simulation) with a number of α-β parameter pairs, one of which is represented in Case B in the preceding discussion. Furthermore, some form of programmed gate increase will be included in BCAS to account for range error buildup during track coasts. However, the exact logic which triggers the increase and the size of that increase is not yet clear. In any case, the same logic would be operative during a false track and would therefore increase its survival probability. The ultimate effect is an increase in the false alarm rates above the values computed in Appendix C. The latter analysis is based on a constant range gate, ±240 ft, and consequently provides only a lower bound on the false-alarm rate.